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British Teachers' Reactions to the Cuisenaire-Gattegno Materials

The Color-Rod approach to Arithmetic

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DURING THE FALL of 1956 a study was made of the Cuisenaire-Gattegno color approach to arithmetic as it was used by several teachers in infant and junior schools in the area of London, England. The purpose of the study was to secure the teachers' reactions as to the effectiveness of the approach under everyday classroom conditions.

During the study, twenty-two classes were observed using the materials, and thirty-one teachers using the color approach were interviewed. A number of classroom demonstrations by Dr. Gattegno were also observed.

The materials used in the Cuisenaire-Gattegno approach consisted principally of a large number of rectangular rods, varying in length from one to ten centimeters, and colored so that certain set relationships of number could be demonstrated. The children, individually or in small groups, manipulated these rods in certain ways in order to "discover," or gain insight, into some of the simple but fundamental mathematical concepts that underlie our number system. Certain charts, involving a game element, were used for practice purposes, and formed part of the materials.

The method of using the materials followed a general procedure that was suggested in the teacher's handbook.[1] The principles underlying the procedure are discussed briefly in Gattegno's article, "New Developments in Arithmetic Teaching in Britain."[2]

As will be noted below, some of the questions the teachers were asked during the interviews were selected to determine whether or not any problems arose in professional or public relations due to their experimenting with the use of color in teaching arithmetic, and the remaining questions were selected to determine the teachers' general impressions regarding the effectiveness of the approach and the materials used. The comparatively small number of cases used in the study, and the difficulties that were involved in obtaining a representative sample did not permit a statistical study of the effectiveness of the procedures. It was thought, nevertheless, that the opinions of the teachers concerned would be of interest to others who might contemplate experimenting with the Cuisenaire-Gattegno materials.

[1] G. Cuisenaire and C. Gattegno, *Numbers in Colour, A New Method of Teaching Arithmetic in Primary Schools* (second edition; London: William Heinemann Ltd., 1953).

[2] C. Gattegno, "New Developments in Arithmetic Teaching in Britain," THE ARITHMETIC TEACHER, 3: 85–89, April, 1956.

The following seventeen questions were asked of each teacher during the interviews. A comment on the teacher's replies follows each question.

QUESTION 1. *Do the children usually seem to enjoy the color approach to arithmetic?*

To this question, every teacher interviewed answered in the affirmative and without hesitation. Evidently they were quite convinced of the general interest of the children in the procedure. During the classroom observations it was noted that the children manipulated the materials with zest and a high degree of interest. This was noticeable at all age levels and may account, to some extent, for the children's success with the procedure.

QUESTION 2. *Are there some pupils who seem to show very little interest?*

The usual reaction of the teachers to this question was to state promptly that there were some children who showed much less interest than others. Answers to further questions on this point indicated that these were usually the slower pupils, and that the procedure, rather than the materials, should be modified for these pupils. Probably further experiments should be conducted to determine the most effective procedure for working with slow learning children.

QUESTION 3. *To what factors do you attribute the children's interest in the color approach?*

The six answers received most frequently, in the order of their frequency, were as follows:

1. The children's interest in manipulating objects
2. The color appeal of the material
3. The various challenging situations presented by the materials
4. The children's curiosity
5. The clarification of certain arithmetical processes in the children's minds
6. The play element involved in using the material

The variety of replies indicated that a many-sided appeal seemed to lie in the use of the material. The fifth most frequently given answer, "The clarification of certain arithmetical processes in the children's minds," seemed significant in the light of other studies that have shown that increasing the understanding of arithmetic will strengthen the retention of learning.[3]

QUESTION 4. *What insights into arithmetic do children appear to gain readily from using the material?*

The seven answers received most often, in order of their frequency, were as follows:

1. The children see numbers as wholes
2. A better understanding of fractions
3. A better understanding of division
4. A better understanding of multiplication
5. A better understanding of large numbers
6. The concept of equivalence
7. The concept of volume

The answers showed that the teachers were impressed by the fact that their pupils grasped some of the more difficult concepts of elementary arithmetic more easily than had been expected.

The most frequently given answer, "Children see numbers as wholes," and the sixth most frequently given answer, "The concept of equivalence," may have been prompted by the emphasis given to these aspects of arithmetic by the teachers' handbook on the use of the materials. These answers are significant also in the light of Piaget's conclusions that these concepts are among the most important in the early stages of grasping the meaning of number.[4] The meaning of fractions, and the process of division and multiplication, answers 3 and 4, were probably made more understandable to the children by the nature of the colored rods themselves and by the procedure recommended for their manipulation.

[3] Charles F. Howard, "Three Methods of Teaching Arithmetic," *California Journal of Educational Research*, 1: 25–29, January, 1950; Russell E. Alkire, "An Experimental Study of the Value of a Meaningful Approach to the Operation of Division with Common Fractions" (Master's thesis, Claremont College, Claremont, California, 1949), 211 p.

[4] Jean Piaget, *The Child's Conception of Number* (London: Routledge and Kegan Paul, Ltd., 1952), chap. 1.

The charts, which formed part of the materials, and lent a game element to memorizing the basic multiplication and division facts, possibly strengthened the pupils' use of the multiplication and division process.

The fifth most frequently received answer, "A better understanding of large numbers," was probably prompted by the emphasis given to grasping the meaning of large numbers through "doubling," which was a procedure recommended in the handbook.

"The concept of volume," the seventh most frequently received answer, may have been a by-product of working with the three dimensional material. Piaget reported that the lack of ability to think in terms of three dimensional material was one of the causes of difficulty during early arithmetical learning.[5]

The writer felt that the concept of proportion was also being developed by the children, as he noted during his observations that they often used elementary ideas of proportion when solving their problems by manipulating the materials, and that the nature of the material lent itself to solving problems through the use of proportion.

QUESTION 5. *Where does the use of numerals come into your plan?*

This question was included because the procedure recommended in the handbook permitted young children, in the first stages of using colored rods, to identify the quantities they were manipulating by color names such as "black," "pink," and so on, rather than by numerals. This did not mean that "black" would stand for "seven" or that any color would "stand for" a number. It meant that at first the child could identify the quantities he was manipulating by such statements as, "Two yellow rods are as long as one orange rod."

The answers to the fifth question, "Where does the use of numerals come into your plan?" indicated that the teachers encouraged the children to use numerals quite early and as they found a need for recording their answers. The use of numerals thus increased gradually as the children grasped the meaning, and as more difficult problems were attempted. The attention of the children was at all times directed primarily to the relationships of the quantities themselves rather than to the names of the quantities, i.e. the numerals.

QUESTION 6. *Do the children experience difficulty in transferring their number ideas to social situations involving arithmetic?*

This question was included because the procedure recommended seemed to put the principal emphasis on the understanding of the number system rather than upon the use of numbers in social situations.

The answers indicated that the teachers were unable to estimate the extent to which children's work with the material carried over to social situations. In all the cases interviewed, the children were being given some practice in the social applications of arithmetic in addition to using the materials. Consequently the teachers were unable to estimate the extent to which each type of experience contributed to the total learning. Unless further experimentation indicated otherwise, it is probably desirable for a teacher to continue to emphasize the social applications of arithmetic along with the use of the Cuisenaire-Gattegno material.

QUESTION 7. *Apart from the use of the Cuisenaire-Gattegno material, what other arithmetical experiences do you give your pupils?*

The answers varied considerably among teachers of different grade levels. As was noted above, all teachers included arithmetical experiences other than those furnished by the material. The social uses of arithmetic, particularly those where money calculations were involved, were emphasized for the older pupils. Counting experiences were most frequently mentioned for the younger children.

[5] *Ibid.*

QUESTION 8. *What amount of time do you give to arithmetic instruction?*

The answers showed that the teachers usually devoted from thirty to forty-five minutes daily to some type of arithmetic instruction. There was no definite proportion of time regularly assigned for the use of the Cuisenaire-Gattegno material, although, with the exception of the practice charts, the material was used more for developing new concepts than for reviews or practice work. The material was often used by the children individually during study periods. The self-correcting aspects of the material made it useful in this respect.

QUESTION 9. *How do you group pupils for instruction?*

The answers showed that, with few exceptions, the teachers had the whole class work together as one group for part of the arithmetic period, and then they had the pupils work in smaller groups, according to their ability, for the remainder of the period. The teachers stated that differences in the pupils' abilities could often be detected quite readily by observing how they manipulated the material in solving their problems during the time they were working in small groups.

QUESTION 10. *Can the pupils work profitably with the materials and without the teacher's direct help for periods of from twenty to thirty minutes?*

The answers were all in the affirmative, although the answers to further questions indicated that the brighter pupils could work alone profitably for longer periods of time than could the slower pupils.

QUESTION 11. *How many pupils can work effectively with one set of colored rods?*

The answers showed that the number was determined by both the age of the pupils and the arithmetical step being studied, but that from two to six pupils usually worked with one box of colored rods and in one group.

QUESTION 12. *How do you evaluate your pupils' progress in arithmetic?*

The teachers of the lower grades usually answered that they evaluated the pupils' work by observing them manipulating the materials, and by examining their written work. The teachers in the upper grades reported that their evaluations were based largely upon the examination of written work.

QUESTION 13. *Have you noted any advantages or disadvantages in using the materials and procedures apart from the mathematical aspect of the work?*

The advantages mentioned by the teachers could be summed up under two points as follows:

1. Some gains in confidence with which pupils attempted new problems were noted. These gains were reported to be more noticeable in the bright pupils.
2. A very favorable opportunity was afforded the teacher for noting and evaluating personality traits of the children, such as persistence, independence, initiative, and so on, as the manipulation of the material afforded a variety of challenging situations of many degrees of difficulty.

No disadvantages were reported by the teachers.

QUESTION 14. *What are the parents' reactions to your use of the colored material in teaching arithmetic?*

Most of the teachers had no evidence of the parents' reactions. No teacher reported unfavorable reactions. Favorable reactions ranged from "Parents very interested" to "Parents approve when they see the children working."

QUESTION 15. *How do the teachers who have not used the material themselves react to your use of it?*

Many of the teachers had no evidence on this point. Replies ranged from "A neutral attitude" to "Interested."

QUESTION 16. *Is a course needed by teachers in order for them to use the material effectively, or are the procedures explained sufficiently in the teachers' manual?*

The majority of the teachers felt that a course was needed. A minority felt that although a course was not necessary it would be very helpful. It was noted that all the teachers interviewed had had some type of course in the use of the color approach.

QUESTION 17. *Have you any general comments you would like to make about the Cuisenaire-Gattegno approach?*

A wide range of responses, all favorable to the approach, showed that the teachers, without exception, were convinced that the approach held considerable promise. The reply, "Very helpful for bright pupils" was repeated in various ways. Replies to the effect that a teacher should not use the approach until he felt he understood its purpose and was familiar with the procedures were often expressed.

Some general conclusions of the writer, drawn from the results of the interviews and from his observation of classroom activities and demonstrations, were as follows:

1. The Cuisenaire-Gattegno color approach to arithmetic has been used by experienced teachers to their satisfaction. There was general agreement that the approach was valuable and held promise for future development through experimentation.
2. While some benefits were available for slower pupils, the average and brighter pupils seemed to benefit particularly by the teachers' use of the approach.
3. The consensus of opinion of the teachers interviewed was that certain mathematical concepts that are not usually developed easily in children by current approaches to arithmetic were facilitated considerably by the use of the material in the recommended manner.
4. At present the Cuisenaire-Gattegno approach holds considerable promise as a supplement to current methods, and further studies should be made to evaluate its effectiveness and to develop the procedures.

EDITOR'S NOTE. We are indeed indebted to Professor Howard for his report on the experience of British teachers with the Cuisenaire-Gattegno materials for learning arithmetic. The basic set of colored rods provides a color association with number which does not seem to impede the development of a sense of number and the use of visual-manipulative materials gives pupils an opportunity to form concepts of size and the relationships of size before these reach the number symbol stage. With rods, the basic concept is linear whereas many teachers believe, through experience, that the collection idea of number is more fundamental to a young child. Professor Howard's report shows that results with the new materials are superior to those of the standard British pattern of instruction which is not described. The novelty and game elements appeal to youngsters and create interest which is necessary to good learning. Children seem to develop independence and persistence which are normal outgrowths of interest.

We need a good "controlled" experiment with the Cuisenaire-Gattegno materials which will compare learning with them in relation to learning by the use of the better materials available in this country so that we may have a measure of their useful contribution here. One must not give total credit to aids and devices when the role of the teacher is an important factor. What a teacher does with children and how she does it and the way she does these things with the materials she uses constitute the program or pattern of education. It is difficult to isolate any of these factors for appraisal. Likewise, although we can measure some of the results of pupil learning, it is difficult to record the mental processes involved in learning.

Visual-Tactual Devices and Their Efficacy

An Experiment in Grade Eight*

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The increasingly technical society in which we exist places upon students of today increased demand for scientific knowledge. A basic tool of all science is mathematics. No one knows how many engineering man-hours went into the production of the Wright Brothers' airplane which made the historic flight of one hundred twenty feet at Kitty Hawk, North Carolina in 1903. We do know that it required 464,000 engineering man-hours to produce a heavy bomber used in World War II. Contrast this with the 10,320,000 engineering man-hours now required to produce a modern heavy jet bomber (1). A basic part of every engineer's education is mathematics. Growth in mathematics is slow. It cannot be hurried. Teachers must attract more students to this subject and then teach those students in the most effective manner.

According to the United States Office of Education, eighty-six per cent of students attending high school in 1900 studied mathematics. By 1950 this had dropped to fifty-five per cent. Since only about eight per cent of all youth of high school age were attending the public high school in 1900 as contrasted with sixty-four per cent in 1950, it is clear that we are providing a high school education for more of our youth. It is cause for concern, however, that in an increasingly technical society relatively fewer students study secondary-school mathematics, the basic tool of all technical subjects (2).

Few will dispute the importance of arithmetic as part of the early schooling of all children. Norman Cousins sums it thus:

> There has never been any argument among educators concerning the importance of the three R's as fundamental building blocks in early schooling. There has, however, been some difference of opinion concerning the methods by which the three R's should be taught (3).

There is a growing body of opinion that the way to improve the learning of arithmetic is to teach it with the goal of understanding by the pupil. Understanding implies the ability to build generalized concepts by organizing particular facts. Schools cannot anticipate all the problems that students will need to solve. They must therefore teach in such a manner that the necessary facts will be retained, understood, and transferred to new situations. "Understanding is the first and foremost prop in retention" (4).

It is not difficult to find opinions that visual aids will assist in the attainment of understanding and retention. We cite a few such endorsements:

> Although much research is needed on the effectiveness of instructional materials on learning, the teacher can feel confident that selection and use of many and varied materials will bring about increased learnings in the social and mathematical aims of arithmetic (5).
>
> An increased use of models would make the learning of slow-learners easier if studies of the characteristics of these learners have any validity (6).
>
> The use of multi-sensory aids in the teaching of pupils with average intelligence or lower is unquestionably necessary. . . . This does not mean that pupils of superior ability do not need to use these sensory aids (7).

Little research is available to support these opinions. Perhaps these testimonials are based upon what might be called "face-validity." That is to say, the aids are created to help present those abstract concepts which are known to give difficulty to students. After their use teachers have a feeling these aids have been helpful and so continue to use them.

* This article is based upon research done recently at the Pennsylvania State University and submitted as a doctoral dissertation.

The use of training aids by the Armed Forces during World War II likely exceeded in extent any past or present civilian use. However, almost no research was conducted to determine the effectiveness of these aids. A curious paradox may be observed. The Armed Forces assumed that "the superior effectiveness of motion pictures, models, and other visual and aural devices had been well documented by civilian education" (8). Since the war schoolmen have assumed the value of visual aids had been well documented by the Armed Forces.

Statement of the Problem

The main purpose of this study was to learn what effect, if any, a kit of sixteen visual-tactual devices had upon the learning of a unit involving areas, volumes, and the Pythagorean relationship in eighth grade arithmetic. Three sub-problems were also investigated. These were: (1) Do dull students profit relatively more or less from the use of visual-tactual devices than do bright students? (2) Is there a relation between the amount students use the devices and their achievement in arithmetic? (3) Do these devices alter students' attitudes toward mathematics?

Student Population

The population involved in the study was composed of all the eighth grade students in three suburban junior high schools all located within five miles of the city of Lancaster, Pennsylvania. The five hundred forty-one students were distributed among eighteen sections, an average of approximately thirty students per section. Nine of these sections, taught by three teachers, comprised the experimental group. The remaining nine sections, taught by two teachers, made up the control group.

Selection and Instruction of Teachers

An effort was made to select teachers who were nearly equal in ability and who were willing to teach under the conditions imposed by the study. Final selection was made by the writer and two other colleagues from the Mathematics Department of Millersville State Teachers College. Teaching ability could not be the sole criterion of selection. Teachers of the experimental groups needed to be sympathetic to the idea of using visual aids in teaching arithmetic. Also it was desirable that they should have had some experience in using these aids. The three teachers of the experimental groups had studied mathematics in college classes taught by the writer. Since visual devices are often used in these classes and students are required to make some of them, it was thought these three teachers would be familiar with the visual aids and the technique of using them. In order to be certain of this, several meetings were held to familiarize the teachers with the devices and each was supplied with written instructions for using each device.

The two teachers of the control groups had not used many devices in teaching arithmetic. Their experience, rapport with students, enthusiasm for the experiment, and classroom technique were such that all three persons rating them agreed they were competent teachers.

During the first six weeks of the semester instruction was concerned with review materials largely computational in nature. During this time no attempt was made to differentiate instruction. All students were supplied with the same textbook (9). The actual experimental period was the eight weeks immediately following this review.

The Testing Program

The five tests were administered as noted. With the exception of the Mental Ability Test, all tests were of forty-five minutes duration.

The tests on Surfaces and Solids were prepared by the author. The following procedure was used in developing these tests. Sample problems were submitted to four competent judges. After consideration of the constructive criticism of these four teachers a sample test was prepared. This test was

Name of test	Length	Reliability*	When given
Otis Mental Ability	30 minutes		Second week of review period
Progress test	40 items	0.87	End of six weeks review
Surfaces	30 items	0.87	End of 3½ weeks experimental inst.
Solids and right triangles	26 items	0.89	End of 4½ weeks additional inst.
Retention test	28 items	0.89	Twelve weeks after end of experimental teaching.

*Kuder-Richardson formula #20 used to determine these reliabilities.

administered to an average ninth grade class in a school not participating in the experiment. This permitted rewording certain problems and establishing time limits. An attempt was made to frame the test items so as to require understanding as well as recall. For example, problem twenty-seven on the Surfaces test was: "The area of a triangle is the same as the area of a certain square. Each has a base of ten inches. What is the height of the triangle?"

The Retention test was composed of items selected from both the tests dealing with surfaces and solids.

Questionnaires for Additional Information

The first questionnaire asked students to indicate their preference for the four major subjects studied during the year. Students responded to this questionnaire during the second week of school and again after the conclusion of the experimental teaching. The purpose of the repetition was to determine if there was any change of attitude toward mathematics during the study. In order to avoid bias for or against any subject the questionnaires were administered during guidance classes by guidance teachers. Students were not told the questionnaires were in any way related to the study.

A second questionnaire, administered by the experimental teachers to experimental classes only, attempted to determine to what degree the students found the devices helpful and also to learn to what extent the students actually used the devices themselves. This was administered after the conclusion of the experimental teaching.

Procedure and Experimental Results

All teachers were asked to teach for understanding. The chief difference in instruction was that the experimental teachers used a kit of sixteen visual-tactual devices in their presentations to the class and these devices were available at all times to students in the experimental classes.

The technique employed was that of matched groups. At the conclusion of the experimental period the attendance records of all students were examined. The data from all students with three or more days of absence were excluded from the statistical analysis. The data for additional students were discarded for other special digressions such as taking tests more than one day late. The remaining students were used for matching. Matching was done on the basis of sex, mental age as indicated by the Otis Quick Scoring Mental Ability test, and the scores made on the Progress test given at the end of the six weeks review period which preceded the teaching of the experimental material. The greatest divergence allowed was five months in mental age and five points on the Progress test. No student pairs were this divergent on both criteria. It was possible to obtain 105 matched pairs of girls and 99 matched pairs of boys. This number of matched pairs composed of 51.5% girls and 48.5% boys corresponded closely as to sex with the 52% girls and 48% boys in the total group studied. While the two groups matched closely on mental age, both the boys and the girls in the control groups had slightly higher mean scores on the Progress test. Thus if there was any

TABLE I

COMPARISON OF MATCHED EXPERIMENTAL AND CONTROL GROUPS

Groups Name of test	Experimental		Control	
	Mean	σ	Mean	σ
Girls $N=105$				
Otis (M.A. months)	171.37	18.10	171.23	18.18
Progress (Scores)	26.46	5.47	26.74	5.41
Boys $N=99$				
Otis (M.A. Months)	169.18	18.80	169.23	18.66
Progress (Scores)	26.39	6.23	26.59	5.55
Total $N=204$				
Otis (M.A. Months)	170.31	18.47	170.26	18.44
Progress (Scores)	26.43	5.85	26.67	5.48

lack of advantage it would seem to fall on the experimental group.

Comparison of Groups on Criterion

The eight weeks of experimental instruction was divided as follows: three and one-half weeks devoted to Surfaces, and four and one-half weeks to Solids and Right Triangles. A test on each of these units of instruction was administered at the conclusion of the unit. The criterion was the sum of the scores made on the two unit tests. Table II shows the comparison of the matched groups on the criterion. While the experimental groups scored higher on the criterion none of these differences are statistically significant.

An attempt was made to determine if students whose mental ability was higher than average or lower than average profited differently from using the visual-tactual devices. The thirty girls with the highest mental ages in the experimental group were compared with the highest thirty girls in the control group. All girls in this "highest" group had a mental age more than 180 months. The mean of this group was approximately one standard deviation above the mean for all girls in the study. Likewise the thirty girls in each group with the lowest mental ages were compared. The mental ages of girls in this lowest group were all less than 160 months. Similar groups were

TABLE II

COMPARISON OF EXPERIMENTAL AND CONTROL GROUPS ON CRITERION

	Mean Raw Score	σ	Diff. of Means Exp.-Control	"t"
Girls $N=105$				
Experimental	27.56	11.94	0.32	0.39
Control	27.24	10.39		
Boys $N=99$				
Experimental	29.42	10.65	1.23	1.16
Control	28.19	11.07		
Boys and Girls				
Experimental	28.47	11.37	0.77	1.15
Control	27.70	10.74		

selected from the boys with the highest group having mental ages above 178 months and the lowest below 156 months. Since there were approximately 100 matched pairs of each sex, selection of thirty students represented about the highest and the lowest thirty per cent. Table III shows the scores made on the criterion by these sub-groups.

From Table III no definite pattern of achievement on the Criterion test is discernible. None of the differences is significant. It is interesting to note, however, that with students of lowest mental ages the observed score of the control groups was higher than that of the experimental. The widely held opinion that students of low mental ability profit more from such visual-tactual devices would appear to be questioned by this result.

TABLE III

	Mean M.A. (months)	σ	Mean Score	Diff. means Exp.-Cont.	"t"
Comparison of Criterion Test Scores of Learners with Relatively High M.A.					
Girls $N=30$					
Experimental	194.1	9.37	39.50	2.00	1.09
Control	193.8	9.49	37.50		
Boys $N=30$					
Experimental	190.9	9.60	37.17	0.03	
Control	191.1	9.89	37.14		
Comparison of Criterion Test Scores of Learners with Relatively Low M.A.					
Girls $N=30$					
Experimental	150.8	7.23	17.87	−0.90	
Control	150.2	6.83	18.77		
Boys $N=30$					
Experimental	147.6	8.43	19.53	−1.77	0.63
Control	147.9	8.63	21.30		

The Retention test of twenty-eight items was administered twelve weeks after the conclusion of experimental teaching. Teachers of all groups avoided any review of the experimental subject matter during the interim. Table IV shows the scores made on the retention test. While none of the differences are significant, the difference for the boys on the Retention test comes closest to being significant. It should be noted that once again the scores of the experimental groups are higher than those of the control groups.

The first questionnaire revealed that students placed mathematics first in preference among the four major subjects studied in eighth grade. This result was rather surprising in the light of the many recent statements made relevant to students' attitudes toward mathematics. The devices had no effect on this rating. While students stated overwhelmingly that they found the devices to be helpful apparently the use of such devices had little effect on preference for mathematics. Moreover, correlation between the amount of use and scores made on the Criterion test were +0.06 for the girls and −0.03 for the boys.

Conclusions

The experimental groups (those using the sixteen devices) consistently scored higher, though not significantly so, on both the Criterion and the Retention tests. It would appear that the devices gave some assistance in learning the arithmetic taught and were a bit more helpful in promoting retention. Visual aids are not a "cure-all" for mathematics ills any more than one of the modern wonder drugs are a "cure-all" for physical ills. Each serves a useful but limited purpose.

TABLE IV
COMPARISON OF EXPERIMENTAL AND CONTROL GROUPS ON RETENTION TEST

	Mean Score	σ	Diff. Means Exp.-Control	"t"
Girls $N=104$				
Experimental	10.24	6.42	0.32	0.57
Control	9.92	5.85		
Boys $N=98$				
Experimental	12.32	5.63	0.84	1.50
Control	11.48	6.58		
Total $N=202$				
Experimental	11.25	6.15	0.57	1.42
Control	10.68	6.28		

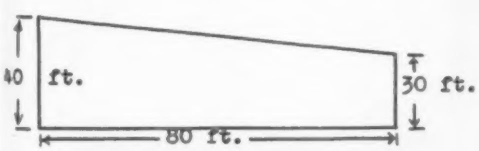
The Retention Test Used in the Experiment*

1. The diameter of a bicycle wheel is 28 inches. What is its circumference?

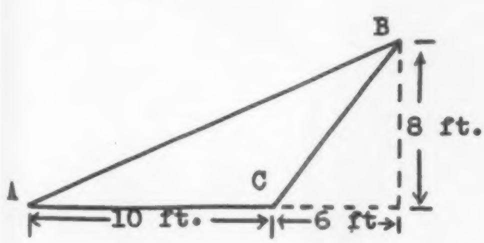
* Spaces for answers which were printed on the original test are not shown here.

2. The altitude of a trapezoid is 12 inches. One of the parallel sides is 18 inches and the other is 14 inches. What is the area of the trapezoid?
3. A rug measures 9 ft. by 12 ft. How many *square yards* are in this rug?
4. How many diameters would be required to go completely around a circle?
5. The area of a circle is how many times the area of a square whose side is equal to the radius of the circle?
6. The perimeter of a certain square is 28 inches. What is the length of one side?
7. In triangle ABC angle A is 78° and angle B is 18°. How large is angle C?

8. The base of a triangle is 14 inches and the height is 10 inches. What is the area of the triangle?
9. The base of a parallelogram is 14 ft. The height is 7½ ft. What is the area of the parallelogram?
10. What fact is missing in order to solve this problem?
    The perimeter of a rectangle is 60 ft. What is the length?
11. A garden is shaped like this. How many square feet are in the garden?



12. The largest possible circle is cut from a square piece of metal 12 inches on a side. How many square inches of metal are not used?
13. In this figure, find the area of triangle ABC.



14. The area of a triangle is the same as the area of a certain square. Each has a base of 10 inches. What is the height of the triangle?
15. A kitchen 9 ft. by 12 ft. is to have the floor covered with tile squares 9 inches on a side. How many pieces of tile will be required?
16. The volume of a cone is ______ as great as the volume of a cylinder of equal height and diameter?
17. How many boxes, one foot on a side, can be packed in a carton 4 ft. long, 2 ft. wide, and 3 ft. deep?
18. A square foot is how many times as great in area as a square inch?
19. Is an area of one square foot necessarily square in shape?
20. A cube 12 inches on a side contains ______ cubic inches?
21. The surface of a sphere is ______ times as great in area as the area of a circle of equal diameter?
22. If e has a value of 4, what is the value of e^2?
23. Find the volume of a spherical balloon that is inflated so that its diameter is 12 inches.
24. How many board feet of lumber are in a plank 8 in. wide, 10 ft. long, and 2 in. thick?
25. A room 14 ft. long and 12 ft. wide is 8 ft. high. The walls and the ceiling are to be painted. If we deduct 65 square feet for windows and doors, how many square feet are to be painted?
26. A rectangular barn door 6 ft. wide and 8 ft. high needs a diagonal brace. How long must the brace be made?
27. What is the lateral area of a cylinder whose altitude is 14 ft. and whose base radius is 4 ft.?
28. An irregularly shaped stone was placed in a cylindrical container partially filled with water. This caused the water level to rise 2 inches. If the diameter of the cylinder was 3 inches, what was the volume of the stone?

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The numbers correspond to numbers in the text.

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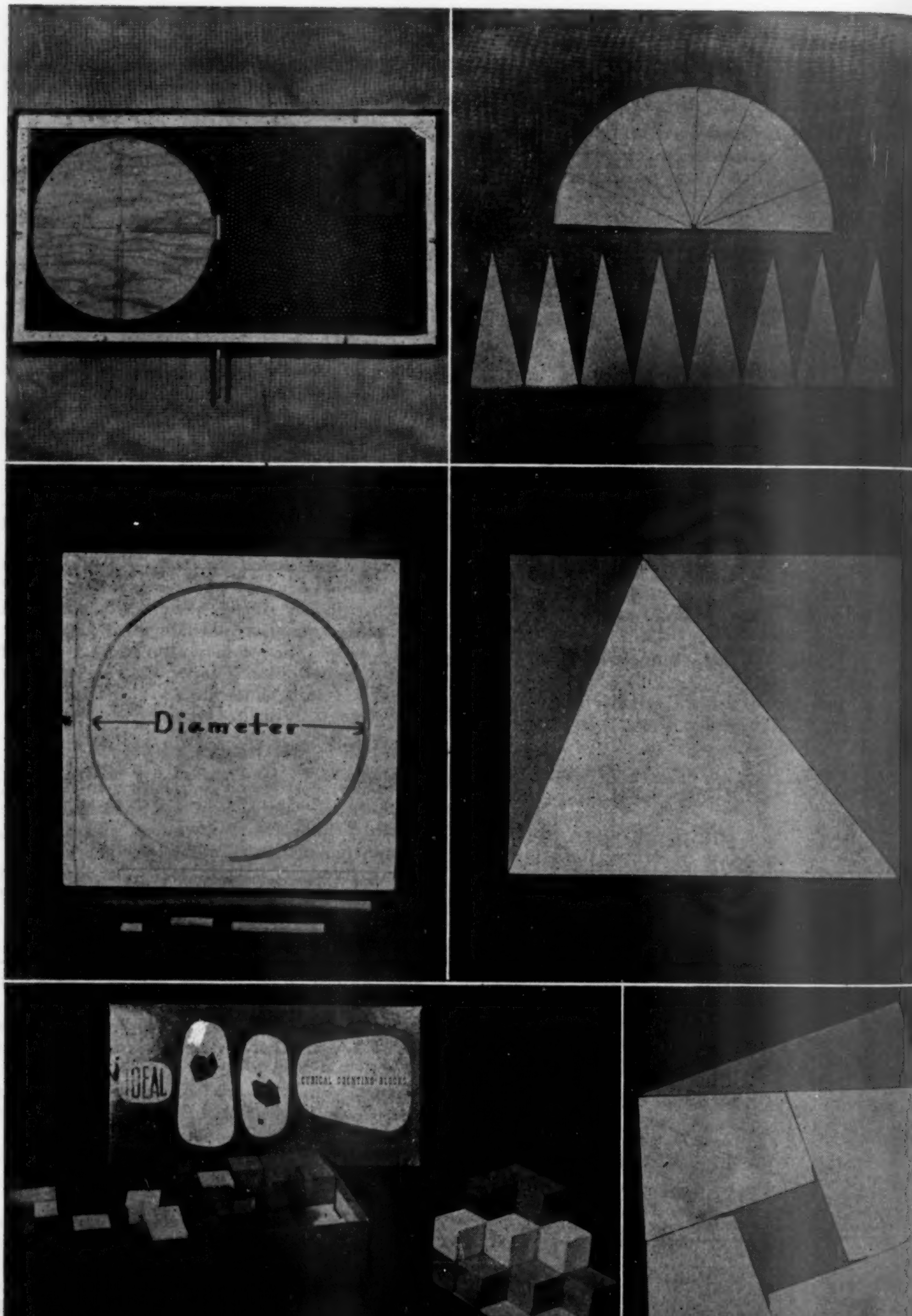
(*Editor's Note on page 203*)



Some of the Multi-Sensory Aids Used in the Experiment

BOOK REVIEW

Arithmetoons, Lowry W. Harding. Wm C. Brown Company, Dubuque, Iowa, 1956. Paper, 91 pp., $1.50.

Each page of concepts and captions written by Lowry W. Harding is illustrated by a cartoon on the opposite or facing page. These cartoons were drawn or designed by Michael Dooley.

The author says that in these busy times few people seem to take time to read about the problems of arithmetic teaching and the education of arithmetic teachers. It is to establish communication with those people who take time only to look but not to read that this book of arithmetic cartoons was designed. It is the hope of the authors that, through this book, people may become aware of the seriousness and complexity of the problems involved in arithmetic instruction.

One cartoon illustrating a father's view on homework shows him working pages and pages of arithmetic examples, his child's assignment no doubt. Another illustrates how results of modern teaching aids depends upon the competence of the teacher who uses them. One teacher using such teaching aids as a projector and screen may still teach no differently (showing multiplication table on the screen) than the teacher of yesterday did in teaching the same multiplication table with a chart on the wall. To illustrate individual differences in ability, attitude and interest, a cartoon shows three children supposedly studying arithmetic—one is reading a comic book placed inside his arithmetic book, another a book on the atom, and the third looking at the pages (seemingly blank to him) of his arithmetic workbook.

One who teaches college classes might like occasionally to post some of these cartoons on the bulletin board to point out through humor some problems involved in education.

AGNES G. GUNDERSON

Visual-Tactual Devices and Their Efficacy

(*Continued from page 201*)

EDITOR'S NOTE. After studying Dr. Anderson's research on the results of having used multi-sensory materials with students in grade eight arithmetic and having noted that he found no definite or marked advantages in favor of learning by these materials one wonders if the *method of use* or the *nature of the testing* or both of these were determining factors in the study. We had assumed that greater gains would have been made by the students. In some studies it has been found that the better-than-average students profited most by the use of visual aids. Let us remember that this study was made at one grade level and with materials largely geometric. We need more studies at other grade levels with materials such as will illuminate the meaning and use of number systems, processes, fractions, etc. Note that Dr. Anderson's test was one of performance and in the main measured understanding of concepts and principles only as these were necessary to the problems. A test that would serve as a "ladder scale" of understanding might shed more light on the values inherent in the experiment. The rate at which perception of geometric concepts and principles takes place should be examined. We are thankful to Dr. Anderson for his experiment. Let us have more before we finally assess the values of multi-sensory aids. In the meantime, the teacher should be the best single aid to the learning of pupils.

New Tools, Methods for their Use, and a New Curriculum in Arithmetic*

Andrew F. Schott
Marquette University, Milwaukee

The arithmetic children learn depends upon the structured learning situations which are a part of their school life and the vicarious learning experiences which occur outside the school. If participation in a structured arithmetic learning situation results in mental frustration or in sheer boredom with the experience future participation in such experiences is either avoided or the learnings resulting from the learning situation are minimal. Motivation is then also absent or minimal. Measurement of learnings in arithmetic appears to be a direct measurement of motivation, not the opposite, as is often supposed. Children who do not learn arithmetic in structured learning situations often report that they "hate" arithmetic. There is an abundance of evidence that learning experiences in arithmetic in school, almost universally, are fearfully frustrating to the majority, and to the gifted, extremely boresome.

This need not be. Children, as Paget has amply demonstrated, are naturally curious and the world of mathematics is fascinating to them in the early years of childhood. Abstractions, generalizations and conceptualizations, commensurate with their level of maturity and ability, intuitively learned and made explicit through understanding and theoretical and social application, challenge their curiosity and generate dynamic learning patterns of self-realization and self-organization. Logical patterns of mathematical thought inherent in arithmetic can be acquired.

To achieve these ends, children's learning experiences in the arithmetic classroom must be structured learning experiences which aid in the development of the abstractions, generalizations and conceptualizations basic to an understanding of arithmetic. The structured learning experiences must be so designed that the child can learn to his maximum potential in a cyclic learning process the more and more complex arithmetic and other mathematics which are needed to meet the demands of our society. These demands have completely changed in the past twenty years. Skills in computation are no longer essential. Calculating machines are available to everyone at prices which are in keeping with their needs. The introduction of the calculating machine has increased rather than lessened the importance of learning arithmetic. The need for extreme skill and speed, however, in obtaining "answers" has disappeared. The demand is now for understanding of arithmetic, its logical structure and its practical social application. A mathematically illiterate member of society is just as bad as one who is linguistically illiterate. Besides, the demand for mathematicians and scientists grows far beyond the presently available supply.

The elementary schools, especially from the kindergarten to the sixth grade, have failed to produce students who have learned arithmetic. To those working in the schools in these grades, this is an obvious fact. Consequently, the program of learning at the higher levels is hopelessly handicapped. Only a few self-taught individuals, or those who had an unusual background, manage to maintain an interest in mathematics as a

* Presented to the American Psychological Association at the meeting in New York City, September 1, 1957.

profession or to gain the necessary mathematical knowledge to enter related scientific fields. There is presently much attention being given to the improvement of the learning of mathematics at the level of the high school and college. Much money is being spent on retraining. The present move is retraining high school teachers to a better level of understanding of modern mathematics. However, the students are less equipped to learn the new mathematics than the old. The teacher struggles to overcome the lack of basic training and finds it almost impossible to teach the new mathematics.

The learning structure of arithmetic at the elementary and junior high school levels has been subservient to the arithmetic textbooks for many years. Mechanistic, repetitive learning structures are still prominent in many of them. Emphasis is still on rote memorization and drill. The textbook types of learning structures in arithmetic have, in the main, resulted in fear, frustration, and boredom. Standardized arithmetic tests, based upon textbook learning structures, have restricted the achievement of students to a low level of mediocrity. The textbook structured learning situation, not by any means an individualized student learning structure, challenges neither the gifted, the average, nor the slow learner to work to the maximum potential which he could achieve.

The Arithmetic Textbook has become *the* tool of learning arithmetic. Until recently, the textbook and pencil and paper were really the only arithmetic learning tools to be found in the elementary school. Arithmetic, as the basis of mathematics, is an abstract science. Structured experience quickly leads to the completely abstract world of arithmetic. Measurement and mass are applied arithmetic—not the science. Therefore, yardsticks, rules, scales and other measuring devices cannot be considered tools for learning arithmetic. They are applications of it. Too often children are taught mathematics by baking a cake instead of learning the mathematics needed to do the measuring.

New Tools for Learning Arithmetic

Arithmetic is a language. It is not, however, the English language. The use of the English language to learn arithmetic, basically the method of procedure found in textbooks, requires that two languages must be learned simultaneously—English and arithmetic. Children learned to understand the simple mathematical language with greater ease and at an earlier age than was thought possible in the textbook—(language dominated)—learning structure when tools of a non-verbal nature were made available to them. These tools as created and designed for use in arithmetic are:

1. The Numberfun Counting Frame for Nursery and Kindergarten, incorporated in the Numberfun Book and Numberfun Game.
2. The Numberaid Abacus—the student and teachers models—and the Numberaid Figure-Pad and Calcuslate for use in grades 1, 2, and 3.
3. The Individualized Arithmetic materials for use in grades 1, 2, and 3, consist of the Acetate Books and the Calcuslate, in which the individualized learning structure is presented to the learner and in which the Numberaid Abacus is an integral part. The Individualized Learning Structure incorporates cyclic teaching, individualized learning which allows gifted, normal and slow learners to proceed at their own learning rate, and classroom organization which permits such individual learning to take place.
4. The Numberaid Abacus Contest, a yearly arithmetic competition among selected first, second, and third grade students of exceptional ability. These contests have been sponsored by the Engineers Society of Milwaukee for the past two years.
5. Standardized Arithmetic Tests as tools for the evaluation of the effectiveness of the learning structure.
6. The Fractionaid and materials for its use in learning structures designed to promote the understanding of the arithmetic of fractions.
7. The Arithmetic Kits—third, fourth, fifth, and sixth grade materials needed to learn how the science of arithmetic is used in measurement and other social and practical applications.
8. The calculating machine in its various forms.
9. The Individualized Arithmetic Materials for grades 6–12, in which the individualized learning structure is presented to the learner and in which calculating machines are integral tools. These materials include learning structures in the application of arithmetic and beginning algebra to the sciences, to business and to the basic needs of citizens in a democratic society.

These tools, learning structures, and materials were created, refined, and designed for production in a research program which was begun in 1951. They are modifications of tools presently used in our society or in related societies. Their use in learning structures utilizes them in new ways instead of the usual commercial applications designed for efficiency, speed, and accuracy.

In accordance with research practice, the tools were selected on the basis of extended search of the literature of arithmetic—sociological, educational, and above all, mathematical.

Promising leads were followed. Much money was spent. Some tools were rejected because of high cost of production. All those in current use were examined and evaluated as they were used in classrooms. The Numberfun Counting Frame, the Numberaid Abacus, the Acetate Materials and Calcuslate, standardized achievement tests, the Numberaid Contest, the Fractionaid and the calculating machine were selected as the most promising tools for use in the desired learning structures to achieve the following ends. Students, at all levels of individual differences, should learn, each to his maximum potential:

1. The language of mathematics as it is used in arithmetic and the English equivalents of the language of mathematics.
2. The mathematical symbols used in the decimal number system, its structure and the mathematical relationships inherent in it.
3. Ordinal and cardinal number.
4. The processes of synthesis and disintegration.
5. The mathematical processes—using the decimal system—i.e. counting, addition, multiplication, subtraction, division, estimation, powers and roots and the algorisms developed for efficient use of such processes.
6. The nature of mathematical proof: Logical patterns of thought.
7. The commutative, associative and distributive laws of arithmetic and algebra.
8. Applications of arithmetic to qualitative and quantitative measurement.
9. The application of arithmetic to social and theoretical problems of the physical, natural and social sciences, to business and to individual social competence.

Four of the eight tools, the Numberfun Counting Frame, the Numberaid Abacus, Standardized Achievements Tests, and the Calculating Machines were ready for testing in the fall of 1954. Some preliminary testing of the Numberfun Abacus, the Numberaid Abacus, the Calculating Machines and Standardized Tests had been done during the 1953–54 school year in grades 4–8 of selected Parochial and Public Schools in the Milwaukee Metropolitan Area with favorable results. Test results on 715 students were the basic data for a preliminary report of findings to June 1954. It was published in 1954 and a report made to the American Educational Research Association in the spring of 1955.

The testing program during the school year 1954–55 was limited to six public, Catholic and Lutheran parochial schools in the Milwaukee Area. The results were summarized in the summer of 1955. The summary was not published. The findings were most encouraging. The Numberfun Counting Frame was developed and incorporated in the Numberfun Book for use with Pre-School and Kindergarten children. The concept of cyclic teaching in which ordinal and cardinal number, addition, multiplication, subtraction, division, problem solving and social application of arithmetic were taught in two five-month cycles each year for the first three years of the primary school, was perfected, as were the concepts of individualization of the learning structure and a classroom learning organization incorporating the new tools and materials.

One thousand, eight hundred and forty-five students in grades 1–12 were tested during the 1954–55 school year. Other types of evaluative data were also obtained.

A more extensive experimental design was outlined in the summer of 1955. Projects in Sacramento, California; in the suburban area of Wilmington, Delaware; in Norfolk County, Virginia; in Culver, Indiana; in Birmingham, Michigan and at the Lapeer State Home and Training School for the mentally handicapped at Lapeer, Michigan, in addition to those in the Milwaukee Area, were established. The Acetate Books were

designed and introduced. The Numberaid Abacus Contests were initiated. Preliminary work was done in the design of the Fractionaid and the materials for the arithmetic tests. Control-experimental groups were established at all levels. Two types of research design were outlined for the next two years of research. The number of students tested during the 1955–56 school year was 3,516. The projects flourished during the school year 1955–56 and 1956–57. The research data to June 1957 is now in the last stages of being statistically evaluated. The proved practicality of the program in the primary grades of the projects has resulted in its use in more schools and school systems in the 1956–57 and 1957–58 school years.

Experimental Results

The statistics available for this report are drawn from those obtained from the Beverly and Pembroke Elementary Schools at Birmingham, Michigan and from some general statistics drawn from the first, second, and third grade statistics from all projects for the 1955–56 and 56–57 school years.

Figures 1 and 2 are bar graphs of achievement in arithmetic from the Beverly and Pembroke Elementary Schools, grades 1, 2, and 3, as measured by the California Arithmetic Test. Figures 1 and 2 indicate the achievement of children in 1st, 2nd, and 3rd grade classes in control and experimental groups over a two year period.

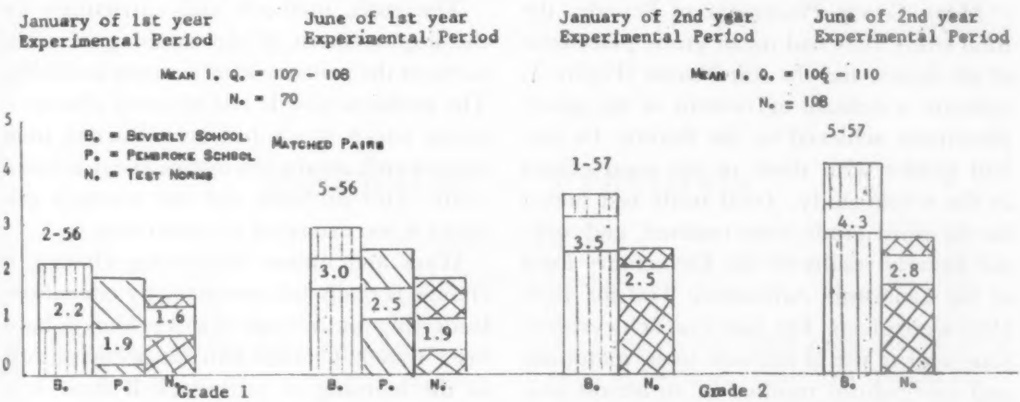


FIG. 1. Grade Placement Total Scores on California Arithmetic Test.

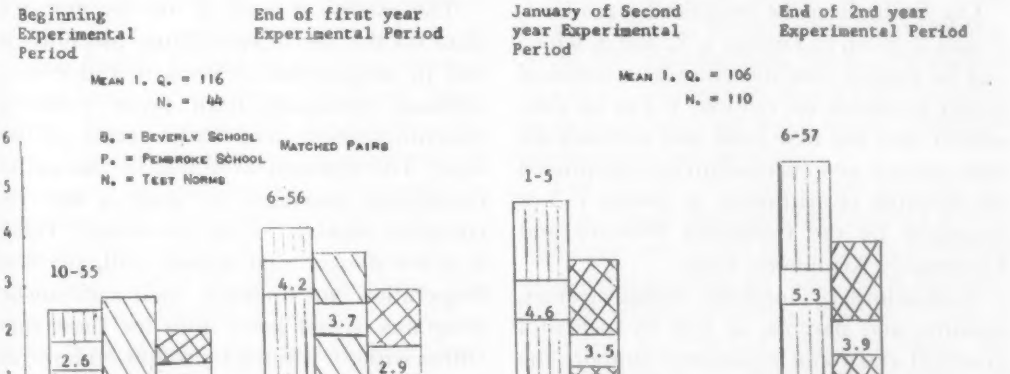


FIG. 2. Grade Placement Total Scores for Grades Two and Three.

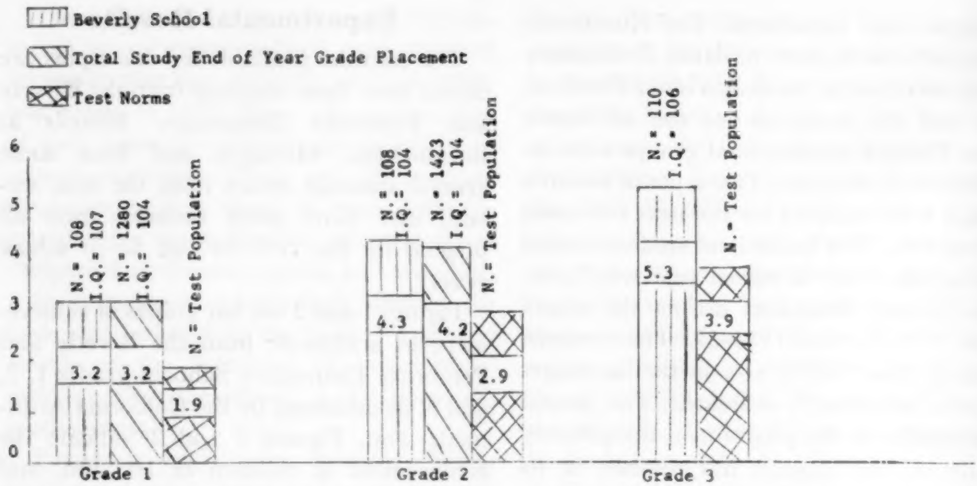


FIG. 3. Grade Placement Total Scores on California Arithmetic Test.

Mean Grade Placement of Beverly, the total study year end mean grade placement of all classes, and the test Norms (Figure 3) indicate a definite agreement of the grade placement achieved by the Beverly 1st and 2nd grades with those of the total classes in the whole study. Total study test norms for the third grade were omitted, and only the Beverly results on the Elementary form of the California Arithmetic Test for June 1957 were given. The year end achievement (one year of use of the new tools, materials, and curriculum) resulted in an almost uniform gain of one year and three months beyond the test norms.

On the basis of the statistics for grades 1, 2, and 3, given in Figures 1, 2, and 3, which will be worked into the complete statistical report available on request, it can be concluded that the new tools and methods for their use in a new curriculum have improved the learning of arithmetic in grades 1–3 as measured by the California Primary and Elementary Arithmetic Tests.

Evaluations by teachers, administrators, children and parents, as well as successful practical classroom experience support the statistical evidence. The fact that the tests are not nearly comprehensive enough to sample the breadth of the learnings of these children is evident in the exhaustion of the test at the third grade level.

The tools, methods and curriculum for the improvement of the teaching of arithmetic at the primary level are now available. The problem now is one of social change—about which much has been learned from the research conducted during the past seven years. This problem and the research evidence is too extended to report here.

Ways and means of creating change, if change is really felt necessary by education-leadership on the basis of this evidence, have been tested. Change can be accomplished in the learning of arithmetic if there is a willingness to accept the research findings as they become available.

The statistical analysis of the research data on the use of calculating machines is still in preparation. Several tentative conclusions, stemming from seven years of experimentation need to be made at this time. The research available in the use of calculating machines to date is not yet complete enough to be conclusive. There is grave danger that schools will, in their desperation to improve their arithmetic programs, spend large sums for those machines without waiting for sound evidence as to the value of such tools of learning. The new arithmetic program being developed in the primary grades shows great promise. On the basis of this promise, the present use of calculating machines is merely reme-

dial. That they will have a place in the developing curriculum is unquestionable. To what extent they will be used, and at what levels, is not yet known. It is better to wait until such research is completed and the usefulness of the machines as teaching tools is statistically validated in a curriculum created and designed for classroom use, supported by programs of in-service training of teachers and teachers in-training.

Editor's Note. Dr. Schott has been busy for several years developing new materials for learning arithmetic and also in using calculators at various stages in the learning process. He is seeking new and better ways for children to learn and to achieve more closely in relation to their abilities. People who are interested in his materials may write to him at 1435 N. 116 St., Wauwatosa, Wisconsin. The results he has reported in this paper show that his experimental groups have surpassed the control group by one year. One asks if this is highly significant. So many factors enter into a learning situation even though the arrangements are "controlled." It is certain however that when one seeks to teach arithmetic and does so with persistence and good learning procedures, the results are usually gratifying. As a case in point, the editor, for several years, took beginning seventh graders who were slightly below national norm standards and by the end of the year achieved results a full year above national norms on standardized tests. Dr. Schott's groups are distinctly above normal in ability. One group is approximately one standard deviation above the normal mean. With this information and with a knowledge of the statistical structure of the test used one could establish an adjusted standard to be used with special groups. Dr. Schott's experimental groups were matched with control groups and hence the difference of achievement is real and in favor of the experimental groups. He has raised the question of adequacy of the testing instrument. He is interested in measuring more factors than appear on tests. No doubt he will have more developmental work to report at a later date.

Reduction of Fractions

Catherine Geary
Middletown, Conn.

Children to whom fractions have been presented in a meaningful manner, know that certain fractions equal, or are the same size as, other fractions. This knowledge has been acquired by the actual division of objects, by using blackboard drawings or the flannel board, by studying fraction charts, and by counting fractional parts. Later, when pupils add simple fractions (like $\frac{1}{4}$ and $\frac{1}{4}$) mentally, someone will always give the answer as $\frac{1}{2}$. This provides the teacher, or another pupil, with the opportunity to state that "2/4 equals, or is the same as, $\frac{1}{2}$." Therefore, when the class is ready for reduction of fractions, a previous knowledge of equal fractions forms the basis for the new concept.

Introducing Reduction of Fractions

The teacher asks the child to divide a paper in two equal parts, and to tell what part of the entire paper each part is. Next a child is asked to separate a second paper in four equal parts, so that he can see that 2/4 of the second paper is the same size as $\frac{1}{2}$ of the first paper. By this means he is led to recall that $2/4=\frac{1}{2}$. Other objective material, like oak tag disks, rulers, and cubical blocks are also used so that pupils can really *see* that fractions are equal to other fractions whose terms are smaller.

The teacher then writes the fraction 2/4 on the board, and asks the children if they can figure out a way to change this fraction to $\frac{1}{2}$ without using an object or a drawing. Pupils enjoy discovering that 2/4 can be changed by dividing the numerator and denominator by 2. Other fractions which can be changed by dividing the terms by the same number are then presented, and pupils figure out how to obtain the answers. Terms, divisible by only one number larger than

1, make this initial reduction easier for the class.

Frequently, some child will offer the information that the preceding process is called "reducing fractions." If no one mentions this fact, the teacher tells the class that "when you change a fraction as we have been doing, we say you have reduced the fraction."

The next question is, "What do you usually think of when you hear the word reduce?"

Some children may say to "reduce speed" but a common answer is to "lose weight" or "get thin." Thus, the concept of reduction as the changing to a smaller form is developed. At the same time, the teacher should emphasize that reduction does not change the size or value of a fraction; it only makes the terms smaller.

In a following lesson, reduction of harder fractions is introduced. When a child reduces a fraction like 12/16 to 6/8, the fact that you can change 6/8 to $\frac{3}{4}$ is demonstrated; and the teacher tells the class that since 3 and 4 can not be divided evenly by any number larger than 1, we say that the fraction has been *reduced to lowest terms.* Board work involving reduction of harder fractions follows. Pupils are reminded to reduce fractions to lowest terms in all answers, and are encouraged to find the largest number that is contained exactly in both terms.

Weakness in division facts, or inability to find the factor to be used, causes difficulty for some pupils. Those who have trouble in reducing fractions seem to find the process much easier if the teacher suggests that they try to reduce a fraction in the following manner.

"Try to divide the numerator and denominator first by the numerator, and then by 4, 3, or 2. With fractions that end in 0 or 5, divide by 5."

With some fractions, like 12/18, children will have to divide twice if they follow this suggestion. They are, however, able to reduce the fraction, and avoid discouragement. As children continue to reduce fractions, they become more proficient in selecting the divisor that will change a fraction to lowest terms.

Teaching Reduction of Fractions

1. Use sufficient manipulative and visual material in introducing reduction of fractions.

2. Allow the pupils time enough to discuss their difficulties. Talking things over not only serves as an emotional release for the children; sometimes it also reveals "hard spots" of which the teacher is unaware.

3. Have the children make up individual study lists of facts missed on a division-fact test, and help them to realize that a knowledge of the 90 division facts is essential for successful reduction of fractions.

4. Make sure that the children understand that when we say "2/4 *equals* $\frac{1}{2}$" we do not mean that the numbers or parts are the same. We mean that 2/4 is the same *size* as $\frac{1}{2}$.

5. Emphasize that when we speak of *reducing fractions to lowest terms* we are not making the fraction smaller—we are only making the terms smaller.

6. Remember that bright pupils may formulate the generalization for reducing fractions easily; allow slower learners to repeat the generalization as a means of clarifying the process in their own minds.

7. Verify reduction of fractions by leading pupils to discover that they can change a reduced fraction to higher terms by multiplying the numerator and denominator by the same number.

Example:

Reduced to lowest terms	Changed to higher terms
$3/6=\frac{1}{2}$	$\frac{3\times 1}{3\times 2}=3/6$

8. Check written work to evaluate learning and reteach when necessary.

EDITOR'S NOTE. Miss Geary shows how to develop and use the concept of reduction of fractions in a meaningful way. This same principle can be used later to establish our conventional procedures in "cancellation" when fractions are used in multiplication. When it seems so human and sensible to establish arithmetical procedures so that children see the sense in what they are doing, one wonders why many teachers still teach by "Rule." The learning of arithmetic can and should be made an educational experience instead of a task of memorization.

The Forgotten Level

Semi-Concrete Materials—A Valuable Bridge

Lillian Packer Drasin
Kinsey School, Philadelphia

Most teachers are aware of the aspects to be considered in the teaching of arithmetic, namely, the logical, social, and psychological. Both the logical steps and the social understandings are defined to a greater or a lesser extent in guides, manuals, and other curriculum sources. When the psychological phase is examined, however, the emphasis is upon technique rather than upon content. Here the teacher's skill, insight, and sympathy toward the learner, his problems and the hurdles he must overcome are of paramount importance.

A prerequisite to the development of such skill and understanding is a recognition of the difficulty of thinking in terms of abstract numbers. All of us realize that the gift of imagination is necessary to the musician, the writer and the artist. How many of us have considered the fact that it requires the very highest type of imaginative projection to think and manipulate in terms of abstract number? Let us examine this further.

The imaginative fancies of children, as those of adults, are primarily based upon previous experiences so reconstructed and re-arranged as to create a completely new picture. Take, for instance, the concept of a fairy. The child is asked to think of the human form, shrink it down to desired smallness, add gossamer wings from a bee, a dragon fly or any flying creature in his experience, and presto! he has an imaginative adventure!

The case for abstract number, however, is an entirely different one. We begin with things. To them, we add the concept of quantity. The things, which are part of the child's experience background, are removed. The abstract quantity is left for the child to manipulate mentally. This is what happens each time a child responds to a number fact or to a given algorism. The following anecdote illustrates the difficulty in gaining this quantitative dimension.

In a first grade room, early in the year, the teacher was telling the children a story about "bunnies." She dramatically presented the existence of five of these little animals, then she introduced the action, namely two bunnies hopped away. "Now," she asked impressively, "How many bunnies were left?" A little doe-eyed fellow waved his hand eagerly. "Yes, Danny," said the teacher, expecting the answer to her problem. "Where did the two bunnies go?" came the unexpected question.

Obviously the quantitative change from the original situation was of little concern to this child. He replied in terms of his previous thinking; therefore many experiences with emphasis upon *how much, how many*, are needed by this child before he can concentrate upon the quantitative *changes* that occur in a problem.

Concrete to Semi-Concrete

We begin, therefore, with many concrete experiences, staying at this level only as long as it is necessary. At this stage the actual manipulation of materials, such as blocks, beads, sticks, etc., helps the child to see what is meant by quantity and what changes occur in re-grouping. There is danger in re- what
maining too long upon this level as well as in going on too quickly. Then again, a frequent practice among teachers is to move directly into the abstract level. The forgotten level, the semi-concrete, plays an important role in providing a means of developing the skill of "mental manipulation."

What is this semi-concrete level? It is that level in which quantity is shown in two dimensional representative materials. At the beginning of this level of learning, pictures of all kinds are indicated. In order to guide the pupils to a more abstract level, geometric shapes are used. Here the emphasis is shifted almost entirely to "how many" rather than what objects are shown.

The *Philadelphia Guide in Arithmetic* suggests varied spot arrangements of numbers from three to nine.*

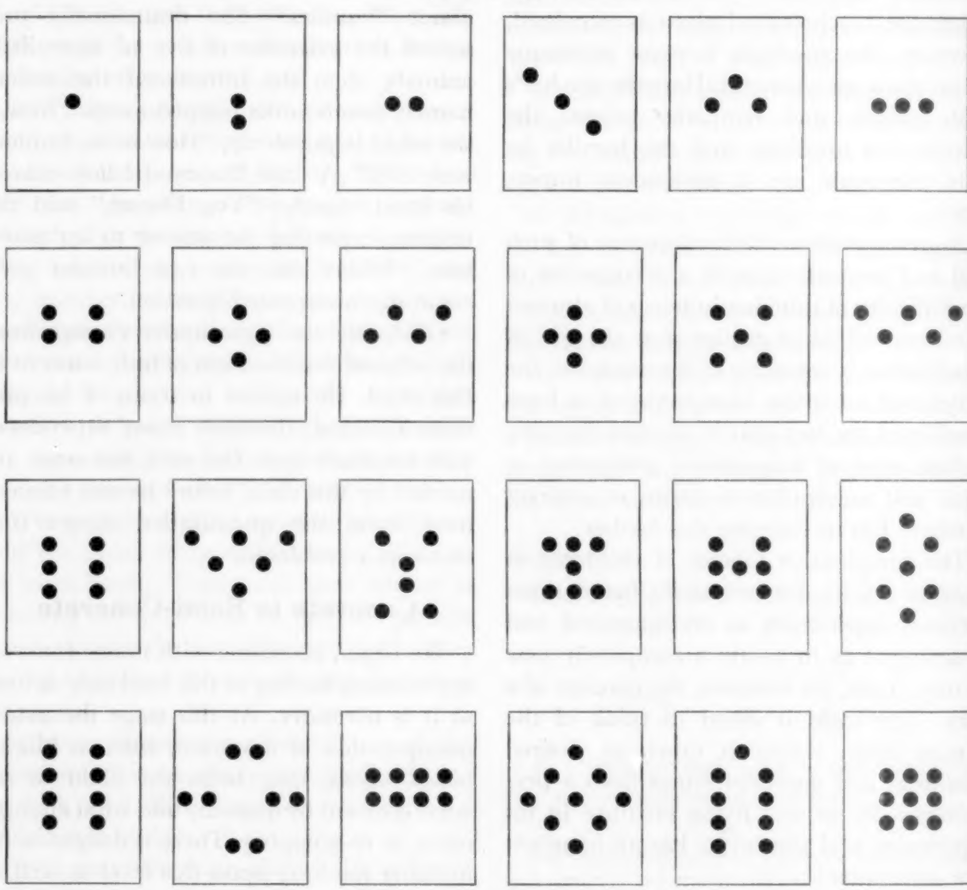
* The first three arrangements are not found in the *Philadelphia Guide.*

The writer has found these spot cards useful in developing group recognition in grades one to three. If a teacher complains of children continuing to use their fingers during arithmetical computation these spot cards have proved helpful in overcoming the use of this crutch, at any grade level.

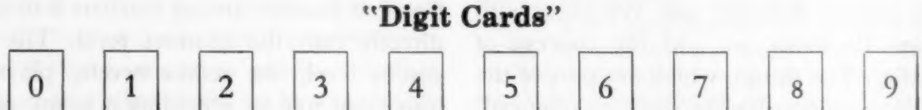
By providing the children in a group or a class with a set of digit cards* from 0 to 9, response is facilitated and every child in the group is involved.

* These cards may be about 2½ by 3 inches in size.

"Spot Cards"

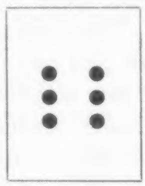


"Digit Cards"



The teacher flashes the spot card. The children respond with the correct digit card. At first much experience with 3, 4, and 5 spots are provided. When the children become proficient and instantly recognize these groups the teacher than proceeds to the larger numbers, helping the children to see parts of groups and how these are put together.

Specifically, the teacher holds up→



The children respond with "6." She then says, "How did you see six?" One child may say, as he points to the spots, "I saw three and three." Another child may say, "I saw two and two and two." A third child may say, "I saw four and two."

If the teacher wishes to lay the foundation for multiplication she says, "How many threes do you see?" The child says, "I see two threes" or "I saw three twos." Much practice at this level helps the child to recognize size of groups and develops an understanding of quantity.

Children may also be encouraged to make their own spot cards to illustrate a process, or help to discover a new number fact. This aspect of work at the semi-concrete level lends itself well to correlation with art. Stick prints, spool prints or cut-outs may be arranged in groups that further emphasize arithmetic learnings.

A large cardboard with an arrangement of ten rows of ten spots is useful in showing related facts. By using two sheets of cardboard as covers, many different sections of the card can be isolated for study so that different grouping and arrangements of numbers are visible. The teacher can do some of this but the pupils should also use the ten-by-ten dot card for study and discovery.

By blocking out with plain pieces of cardboard all the spots except the twelve in the upper corner the teacher can develop the related concepts of

$$4\times 3=12 \quad 3\times 4=12 \quad 3\overline{)12} \quad 4\overline{)12}$$

It is also possible to use the same material to test a child's understanding of numbers. The teacher says, "Show us five 4's. Tell us what other related facts you have shown."

The semi-concrete material can be developed with the class as part of a lesson then referred to for review or later reference.

Whenever such material occurs in texts and workbooks (and it is being used more extensively) the teacher should make use of it at every available opportunity.

Above all, whether the material be teacher or pupil made, it should be displayed so that it is available for ready referral and so that it is looked at repeatedly by boys and girls, thus unconsciously becoming a mental reference.

Summary

In summarizing the "forgotten level," that is the semi-concrete level, we recognize the following:

1. It is the bridge between the concrete and the abstract.
2. It helps the child to concentrate upon the concept of quantity.
3. It is helpful in raising the maturity level of the child from counting by ones to counting by groups.
4. It is a means of strengthening concepts of process.
5. It is a means of testing understanding, particularly in the case of the bright child who can move more rapidly into the abstract level.
6. It is useful in review.
7. It provides a background of material and experience to aid in developing the ability to "manipulate mentally."
8. It is helpful in establishing related facts.
9. It should be displayed so as to provide a ready reference which will consciously or unconsciously become part of the pupil's thinking.

EDITOR'S NOTE. Mrs. Drasin points out a valuable idea in the use of visual and manipulative materials. They are good not only for the static picture they present but also and perhaps often more important, they can be used to show action or change in the reforming of items into other groups. Then too, the real meaning of elementary addition, subtraction, multiplication, and division can be pictured as a process. But it is also important to recall that it is the role of the teacher in organizing and stimulating and leading that provides the real essence of education. The materials themselves without this guidance of the teacher with her pupils is usually of little value. The pupil who has been given a lead, often will proceed far by himself.

Manipulative Devices in Lower Grades

PAULINE HERTZ
New Plymouth, Idaho

IT IS EVIDENT that elementary teachers DO want to improve their teaching of arithmetic. A questionnaire sent to a cross section of the schools in California, Idaho, and Oregon showed 85.4 per cent of those responding to believe that teachers should have more training in the use of manipulative devices in the teaching of arithmetic. Over 36 per cent of these thought the colleges should teach this phase of methods in workshops. And 41.8 per cent believe the training should be received in workshops combined with training elsewhere.[1]

As a result of the above survey, the College of Idaho offered a summer workshop in the teaching of arithmetic. The response was so great that two divisions of the workshop were given and the registration closed. Each time this course is offered it is impossible to accommodate all who wish to register. Elementary teachers feel the need to improve in their teaching of arithmetic.

In this workshop the teacher is shown many commercial devices and their uses. Other devices are made in the workshop, but the emphasis is placed upon the usefulness of each device, when it can be used most helpfully, and when it should no longer be used.

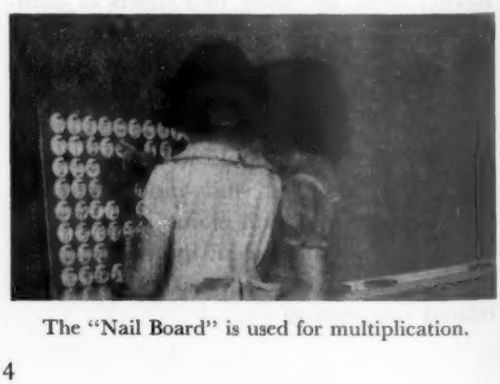
From actual classroom experience and from contact with the above mentioned workshops I find there are definite uses for such devices. First, they help the teacher demonstrate the meaning of the new concept and can be used by the child as he practices. That is, he uses the device as he develops his understanding. Secondly, he continues to use the device as he works for ease in computation of his problem; and finally, he uses the device for practice to gain speed and accuracy in recall.

There are devices which can be used best for just one of these; such as, introducing a new number concept or for developing a clearer understanding of an arithmetical principle. Other devices can best be used for the final stages of drill; that is, drill for automatic recall after the meaning is clearly understood.

Arithmetic Materials Important

Because many of our school officials still fail to understand that arithmetic materials are just as important as materials for art, music, physical education, etc., it is often necessary for the teacher to make or buy her own devices. When this is the case, it is well to know a few devices which can help in the teaching of many things. The "Nail Board" is such a device as it can be used in all the ways that the Hundred Board, the Spool Board, and the Peg Board can be used. It can be used in the primary grades, the intermediate grades, and in junior high school. This device can be made by the teacher at home or by the children at school.

In general a good size for this board is a piece of plywood 22″ by 22″ but a larger size could be used in kindergarten to accommodate larger "spools." On the board



The "Nail Board" is used for multiplication.

[1] Pauline Hertz, "A Study of the Use of Manipulative Devices in the Teaching of Arithmetic in the Primary and Elementary Grades," Unpublished Thesis. The College of Idaho, Caldwell, 1956.

22" by 22", places are marked for ten rows of nails with ten in each row. These will be two inches apart. The board can be painted or varnished. One can use finishing nails or screws. For primary grades where spools are used the screws should be about one and one-fourth inches long. Half inch screws are long enough where cardboard discs are used.

Cardboard discs one and three-fourths inches in diameter with a hole near the edge can be made by the children. Cardboard squares cut on the paper cutter can be used just as well. Milk bottle tops or price tags can also be used. Such tags cost approximately 40¢ a hundred. One set of 100 discs can be marked with the figures one through one hundred. These are of most value in the lower grades for consecutive counting. A plain set, or the reverse side of the numbered discs, can be used at any grade level for such uses as spools can be used in the primary grades or the Hundred Board can be used in the upper grades. In the fourth grade we use the plain discs to show how to add by completing the tens, that six eights are the same as eight sixes, how to find the product by completing the tens, and other similar uses.

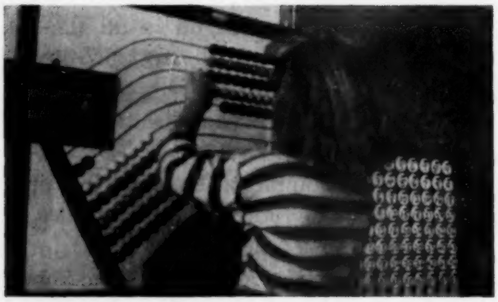
At this grade level where the multiplication facts are learned, we have another set of discs showing the products. The top row and the row down the left side show figures 1–10 in red. (Or the corner disc can be left plain and the others show 1–9 in red.) The other discs have figures in black showing the products. At first we build it as we learn the multiplication facts. After all the multiplication facts through 9 times 9 have been studied, the board is used for drill by some of the children and as a crutch for others.

As a drill for quick recall one or two of the children will remove all the "product discs," leaving the red guides, and see how quickly they can return the discs to the correct places. As you can see, this involves multiplication, division, and factoring if a child mixes the discs and takes them as they come. Some of the "not so bright" children will hunt for certain discs and build back in order; that is, they will fill in all the 2's, then the 3's, etc., until all are back on the board. The child who works in this manner is pleased with his accomplishment which has been at his own level.

We use the completed board as a crutch when we start division. To prevent overuse a child who must refer to the board for a certain combination more than once knows that he should write the fact down and learn it. For the child who cannot learn the multiplication facts, the crutch helps him work along with his peers without a feeling of frustration. He works more slowly because he is using a crutch, but he is doing the same kind of work as the rest of the class.

Pupils at work in the Arithmetic Corner.

An arithmetic corner supplied with devices on various maturity levels, where the children can get help when they need it or find games which are fun as well as drill, is a "device" in itself. Although the children should have freedom in the use of devices, it is well for the teacher to watch for the child who uses a device for a crutch to see that he does not continue that use beyond the point where it should no longer be used as such. For example, the child who uses the nail board as a crutch because he doesn't know his multiplication facts can use the same device for drill in learning them by removing the discs and returning them to the board. The first time he can have someone time him or he can check his starting and stopping time by the room clock. Later he can use the "minute timer" setting it each time a minute or so earlier to see if he can beat his own time.



"The Thinker" is used to show that six eights equal eight sixes

If possible it is well to use more than one device to introduce a new phase of arithmetic. One child might understand one device more clearly than another. The teacher herself might be able to explain more clearly with one device than with another. But one device well used by the teacher and the children is better than many devices which are stored away and used only occasionally or used in such a way that the child and teacher are both confused.

The adhesive boards, especially the flannel board, have long been used by primary teachers. They can be used just as successfully by upper grade teachers for clarifying fraction concepts. A fraction kit made from felt can be obtained commercially or can be made by the teacher from felt, flannel, or Combo. As one fractional part can be placed upon another, division of fractions can be made very meaningful to the child before the rule is developed. Just as the fractions are manipulated on the flannel board, cardboard or paper fractional parts can be manipulated by the child at his desk. Thus the fraction kit can be used to introduce and make meaningful the steps in computing with fractions. It can also be used to prove concretely after a problem has been solved on the abstract level by algorisms.

In conclusion, there are many useful manipulative devices that can be obtained commercially, but if it is impossible for a teacher to get a commercial device, there are also useful and helpful devices which the children can make at school. When the parents and school officials understand the value of these devices, then appropriate commercial devices can be obtained more easily.

EDITOR'S NOTE. Miss Hertz lends her support to the use of visual-manipulative materials and bases this on her thesis study. In reading her article note that she makes it plain that the teacher must do a good deal of planning and as she says "But one device well used by the teacher and the children is better than many devices which are stored away and used only occasionally or used in such a way that child and teacher are both confused." Perhaps the time has come in many schools to spend more time on thinking about how supplementary aid should be used rather than on seeking more gadgets to "show off" to parents and school officials. In other schools visual-manipulative materials are woefully lacking. Good judgment tempered with experience seems to be needed.

An Old Chestnut in Division

Seven boys agreed to work together and share their earnings. When they had accumulated $28, their leader took out his one-seventh share of $13 and with paper and pencil showed the others that he was correct.

Division:	13	Proofs:		13	Add them: 3+3+3
7 into 2, wont go.	7) 28			13	+3+3+3+3=21
7 into 8 is 1, then	7		13	13	+1+1+1+1
multiply, subtract,	21		×7	13	+1+1+1=28.
bring down 2, and	21		21	13	
then 7 into 21 is 3.	—		7	13	
			28	13	
				28	

A Place-Value Game for First Graders[1]

A Teacher-Made Device

Irene R. MacRae

Many teachers find that place value is one of the more difficult number concepts to explain to small children. The following game was devised to help reinforce the understanding of place value after it has been introduced by the use of the *Hundreds, Tens* and *Ones* Chart. This chart is familiar to primary teachers. The game uses the same pocket device used in the chart. Thus children can reinforce their initial learning by practice under conditions similar to those in which the learning took place.

The game can be used in the first grade, but can also be adapted to the second and third grades merely by increasing the size of the numbers set as goals. If the goal is set from 25 to 50 for first graders, it is recommended that 2 to 4 children play, depending on the time available. The more players, the longer the game will take. If the goal is set at 100 for second graders, and 1000 for third graders, perhaps two players would be more practical. The game is easily and quickly made, so that a teacher could provide several games for a class. Children could even make the game themselves.

[1] A Chicago Teachers College Student Project.

Directions

Object of the Game:

To score a predetermined number of points.
For first graders: 25–50 points
For second graders: 100 points
For third graders: 1000 points

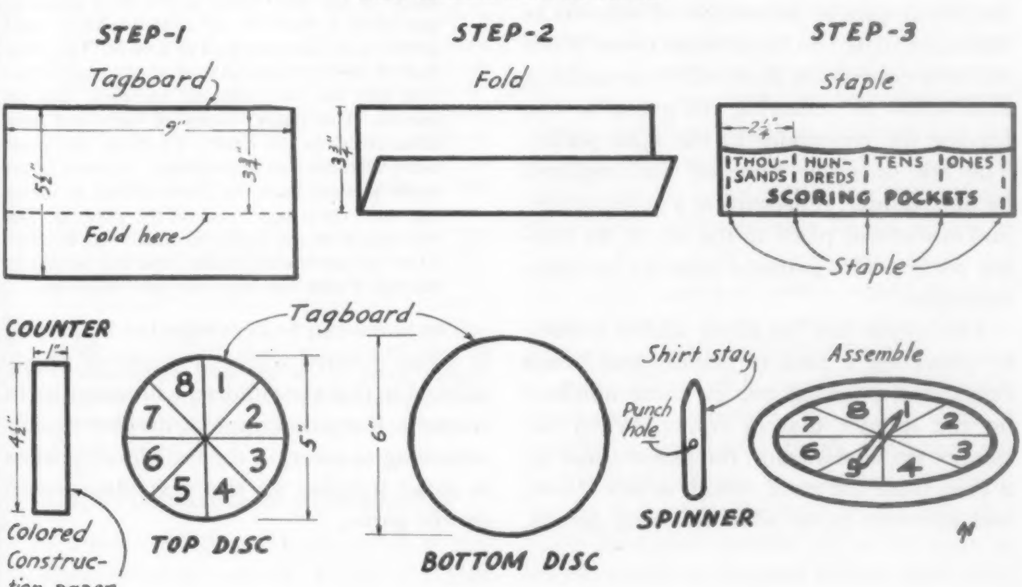
Number of Players:

First grade: 2–4
Second grade: 2
Third grade: 2
Spin to determine who plays first and play to the left.

Procedure:

Each player places the counters in piles in front of him, arranging the colors in the same order as they are placed in the pockets: blue at extreme

The Game in Pictures



right and proceeding to the left, green, red and yellow.

First player spins and places in the extreme right pocket a number of blue counters equal to the number to which the spinner points. The other players do the same. On the second spin, if a player picks up enough more blue counters to equal ten or more when added to what he already has, he "bundles up" ten blue counters, placing them with the pile of blue counters in front of him, and takes one green counter which he places in the next pocket to the left. This continues until some player has accumulated ten green counters in the ten's pocket which he then removes and puts back in the pile in front of him. He then may place one red counter in the hundreds pocket. The first player to do this wins the game unless the goal is set at 1000.

Since the game is one of chance rather than skill, a fast learner may be paired with a slow learner without doing injustice to the fast learner. He can help the slower child and at the same time reinforce his own learning as well as get fun from the game.

There are other values to be derived from this game in addition to the understanding of place value. When used in the first grade, it will serve to give practice in number value as a result of spinning to determine who plays first. It will provide occasion for children to reproduce abstract numbers with concrete counters. To those beyond the immature level of counting, there will be practice in addition by adding the number which the player spins to the number of counters he has accumulated on his previous turns. When this sum exceeds 10, there will be practice in subtraction by removing the group of ten, leaving the remainder in the ones pocket. The ten ones are bundled and replaced by one counter, representing a group of ten, and moved one place to the left in the scoring pocket. This provides practice in transformation.

The game can be given added interest by providing a pack of penalty and bonus cards, and painting two or three numbers on the spinner dial in colors. When the spinner stops on a color, the player turns up a card from the pack, which is face down, and proceeds to do what the card directs, such as, *lose your next turn*, *take an extra turn*, *add 3 ones*, *you lose three of your ones*, etc.

How to Make the Game

The following detailed instructions are merely suggestions. The teacher can use any materials she has at hand.

1. **To make the scoring pockets, use pieces of tagboard, 5 1/2" by 9".**
2. **Fold to 3 3/4" by 9" so that there is a 1 3/4" pocket across the bottom.**
3. **Place a strip of brightly colored mystic tape along each edge to hold pocket in place. The color helps make the game attractive, but the learning values would not be lost if the pocket were held in place by merely stapling.**
4. **To divide the pockets into four sections representing the place positions, black metal rivets were used which were snapped together but not spread with a hammer. This method allows the pockets to stand out a little so the counters can be easily inserted. If rivets are not readily available, strips of mystic tape might be used, or simple stapling would serve the purpose. The words *Scoring Pocket* were manuscripted across the pocket.**
5. **For counters, construction paper in four colors is needed. Cut strips 1" by 4 1/2". At least 18 counters are needed for all colors except for the color representing the 1000 position. Only one of these is needed. (Only nine counters are really necessary for the 10 and 100 positions.)**
6. **For spinner dials, cut four identical circles from tagboard. Mark two into eight equal parts, and write the numbers from 1 to 8 in the sections.**
7. **The spinners in this particular instance were made of the stays from men's sport shirts by punching a hole in the center with a hand punch and inserting part of a rivet. The other half of the rivet was placed at the back of the dial and the two snapped together, but not spread. The blank circles of tagboard were then glued to the backs. To make the game more attractive in appearance, another larger circle was cut from red poster board to match the red mystic tape used on the pockets. This was glued to the backs to make red borders. (Two spinners were made, but one would be enough if only two children were playing.)**

The game can be kept together by placing in a box covered with gay paper or simply marked so that the children will recognize its contents. Keep the counters, divided equally according to color, in the individual pockets to make it easier for the next players who use the game.

Grouping in Arithmetic in the Normal Classroom

CLAUDE IVIE, LILYBEL GUNN, AND IVON HOLLADAY
Meridian Public Schools, Miss.

AS A PART OF THE CURRICULUM re-study initiated in the Meridian Public Schools in 1955, an experimental revision of the mathematics program, grades 1–14, is under way. Experimental revision in the mathematics area is to be centered around three areas:

1. A study of mathematics requirements in general education.
2. An examination of the newer content suggested for mathematics today.
3. A cautious examination of the methods for meeting individual differences—homogeneous and heterogeneous grouping, acceleration and/or enrichment, and individualizing instruction within a class.

A survey of the mathematics program in the Meridian Public Schools was made. Data obtained were:

1. There was evidence that the achievement levels of the various grades from one through fourteen were at or above the national norm on standardized tests. The range of scores was very narrow being no more than two grades and usually one grade between the highest and the lowest student score.
2. The Iowa Algebra Aptitude Test was given a selected group of the top thirty students from the sixth grade of the nine elementary schools in the city. These scores were compared to the scores of students on a similar test given at the end of the eighth grade. The thirty students in the sixth grade scored as high as those did in the eighth grade with the exception of the very top scores.
3. There was indication from a survey made that the elementary teachers, to a greater degree, and high school teachers in lesser amounts used the normal everyday occurrences in arithmetic, or mathematics, for extending or enriching the abilities of students in mathematics.
4. It seemed that most teachers followed a single textbook as a guide with all students in each grade or in each section progressing generally at the same rate.

Experimentation Procedures

As a result of these data a group of elementary teachers worked during the year 1955–1956 in laying the groundwork for the construction of a course of study in arithmetic in grades 1 through 6. In the summer of 1956 a select group of teachers worked for six weeks to construct a course of study. This course of study was examined in the two weeks pre-school workshop and faculty study was devoted to it by all the elementary schools during the year 1956–1957. In the summer of 1957 a selected group of 22 teachers worked to construct specific action research projects to determine the most effective use (a) (collectively) of the general activities which should occur in the school at each grade level to promote arithmetic understanding and (b) (individually) of specific activities such as the drives, the post office experience, or other enrichment practices which can be used normally in the classroom. At the end of the school year 1957 specific reports will be given on these activities and the ways that they may be used to promote arithmetic skills and understanding.

The group which set up the course of study in the summer of 1956 decided upon a policy for meeting individual differences. This policy was that, in the elementary school, children would be accelerated in the skills areas by at least one grade. Diagnostic tests would be given in the first days of the school year. The teachers would set up in "check lists" skills, particularly from the third grade upward, which would be normally expected in any one year in the elementary grades as indicated from the course of study. Each child would be given this check list and would be allowed to work as rapidly as he could through the check list for his normal grade and through the check list for the grade above or below him. No more than two sessions up to an hour in length would be devoted to this skills practices. General use of common mathematical

situations arising in the school day was planned for enrichment and development of understanding for the whole class. These check lists would be filed in his folder and the teacher of the next year would continue the process. It was further planned that groups would form in the classroom of *their own accord* centered around certain skills. The advantage of this type of procedure was that the child would know his needs, would be working at his own rate of speed, and would do so of his own accord.

Recommendations

Recommendations of the teachers who are participating in this experiment are:

1. All activities of a general nature in which the whole group may participate such as drives, picture money, lunch room money, etc., should be used for developing whole group understandings in arithmetic. These activities would be of the enrichment nature. At the Junior High level, taxation, income tax returns, budgeting, personal accounting, etc., would be encouraged as whole-group activities or units.
2. As in reading there seems to be a necessity for teaching specific skills. The suggestion is made that at least two hours a week would be devoted to teaching simple skills in arithmetic. Each child will have a check list of the skills which are "normally" expected during each grade. This of course will include some review of previous skills. A re-check of various skills at least three or four times during the year must be made. As students move through the check lists, they are checked both orally and through various tests which the teacher has devised. These tests are short and to the point. Children will usually group themselves socially around certain large areas of skills so that the teacher will not normally have more than five groups. It is very evident that teachers will need to have many arithmetic books, many arithmetic work books, and various tests and materials equal somewhat to the number of reading books that would be expected in the room. The general consensus of the teachers using this procedure is that most students have really liked and appreciated arithmetic during the year. There is some evidence from scattered testing that many students have made enormous progress and that the range has been extended. The final confirmation will have to occur during the testing of year 1957–1958 and of course during the year 1958–1959.
3. The process attempted is somewhat similar in nature to the reading groups that have been normal practice in the elementary schools for a number of years. However, it must be noted again that the grouping is flexible and has a social basis. For example, the teacher may say, "I want to help the group who is working in two column addition skills for today." She will work with this group while other groups are working on various activities which they have centered upon. She moves from group to group working during the two hours of the week. Many teachers had difficulty during the year getting the class organized so that all groups were not wanting help at the same time. They usually found that, after a period of time with good planning, they could move from group to group without too much difficulty.
4. The final point for consideration must be that of child psychology. The effect of peer relationships is a factor in child psychology which these teachers, who were experimenting, found really operating. As children move into the upper elementary grades, the effect of peer opinion is extensive. Most children group themselves socially. There will be groups which will work more slowly or more rapidly than others because they want to work along together. Some children even resent the grouping. They would much prefer to work in the class as a whole. In certain sections where more children are of the same ability, working as a whole group may be the most effective procedure. Some groups have children who like to work individually or in small groups and there may be as high as ten different groups. It was extremely difficult to get the very superior children to move far out ahead of the other students. The stigma of "brains" or "egghead" or "bright boy" was enough to cause them to not work as fast as they really could.

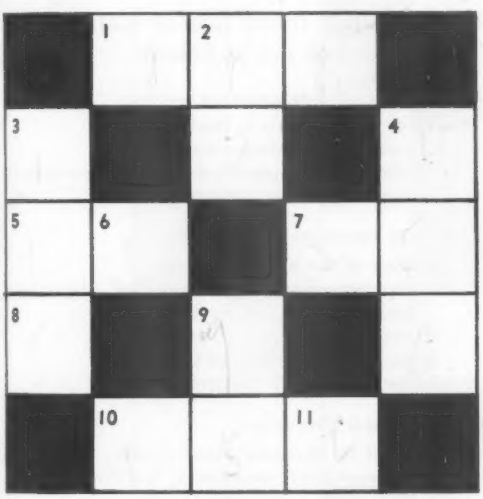
In the effort to accelerate youngsters in the elementary grades, and eventually in the Junior High School, there are a number of problems concerning enrichment and/or acceleration. The present experiment in the Meridian Schools is proceeding on the supposition that there is little difference in acceleration and enrichment as defined in the process above. Further, it is assumed that this process of meeting individual differences will occur in the normal classroom. Eventually there may be separation of children who are mentally retarded or definitely superior.

EDITOR'S NOTE. It is unusual to find so narrow a range of achievement as reported in Meridian. This suggests that lower-level pupils were probably pushed beyond normal limits and that upper-level were not challenged to work to capacity. The new program will undoubtedly produce a wider range

within each class and that is more in harmony with other schools. The new program will seek to develop greater understanding and that calls for modes of teaching and learning that feature discovery and development instead of the memorization of procedures. The pupil who understands the relationships of arithmetic and who can discern a mathematical situation outside a textbook and then knows what to do and how to do it has really mastered arithmetic. It is hoped that the pupils of Meridian will not accelerate at too rapid a rate so that they fail to understand what they are doing and master only the simpler concepts, principles, and skills in the program. The program of enrichment can do much to give life and significance to learning.

A CROSS-NUMBER PUZZLE FOR PRIMARY GRADES

MARGARET F. WILLERDING
San Diego State College

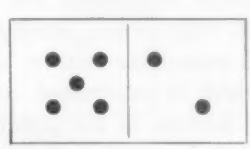


ACROSS

1. What number comes after 110?
3. What time does this clock say?

4. How many tens are there in 40?
5. What time does this clock say?

7. When counting by 5's what number comes after 20?
9.



This is a number picture of what number?

10. When counting by 2's what number comes after 150?

DOWN

2. How many eggs in a dozen?
3. What number comes after 117?
4. What three numbers come after 3?
7. 1+1=?
9. What number comes after 74?
10. What is 6 take away 5?
11. What is 7 take away 5?

Milwaukee's In-Service Arithmetic Education Program

LILLIAN C. PAUKNER*
Milwaukee, Wisconsin

As A PART OF the in-service education program in the Milwaukee Public Schools, designed to encourage professional study and instructional improvement, a course in the teaching of arithmetic will be offered to intermediate grade teachers. Emphasis will be placed on the sequential program for these grades, teaching techniques, provisions for individual differences, use of manipulative materials, and the evaluation of pupil growth. This will be a workshop type of course, giving teachers opportunities to share successful practices in the teaching of arithmetic and the laboratory experience of trying out these practices in their own classrooms.

Participation in such courses as this carry credit for salary classification purposes. "All salary schedules are established in divisions so that members of the staff may qualify by degree status or by earned units of preparation." Credit allowances are given for travel, work experience, authorship, and professional committee service as well as courses earned in accredited schools. An outstanding feature of the program is the freedom of choice each individual has in planning his program of professional activities.

Enrichment Activities for the Gifted Child

The Mathematics Committee at the elementary school level has considered the problem of providing enrichment activities for the gifted or very bright children. This committee has defined enrichment activities as being those activities that:

1. Extend basic understandings and processes in mathematics
2. Give the pupil added power to handle more difficult aspects of what the class is doing
3. Give opportunities for exploration in the particular area of study
4. Give opportunities to relate mathematics to other areas of study—science, astronomy
5. Apply mathematics to every day living activities over and above regular class work.

To assist the teacher in finding activities which will interest the gifted pupil and challenge his thinking, the Committee has summarized suggestions from its thinking and from literature in the field of mathematics. The suggestions are as follows:

EXPERIMENT OR EXPLORATION
- —Short cut methods in computation
- —Discovery of mathematical processes
- —Mathematics implications in science—astronomy, expansion, etc.

RESEARCH PROJECTS AND REPORTS
- —History of the number system
- —Comparative studies—sizes of buildings, bridges, countries, rivers, etc.
- —History of measures
- —Measurement systems of the world
- —Budgets and their construction

CONSTRUCTION PROJECTS
- —Mobiles to illustrate plane figures, solids, etc.
- —Scale models of community, buildings, rooms, etc.
- —Blueprint reading in the construction of models
- —Geometric objects to show cubic measure

MEASUREMENT PROJECTS
- —Laying out garden plots, play fields, etc.
- —Gas and electric meters
- —Measurements in the home

MATHEMATICAL PUZZLES AND GAMES
- —Magic squares
- —Crossword puzzles involving mathematical terms and vocabulary
- —Puzzle problems

DRAWINGS
- —Geometric designs
- —Graphs to illustrate statistical information

FIELD TRIPS
- —Places where mathematics is important—banks, weather bureau, etc.
- —Mathematics implications in other field trips (gifted children alerted to look for them)

* Director of Upper Elementary Curriculum and Instruction (Grades: 4–8). Milwaukee Public Schools.

SCRAPBOOKS

—Pictures, news items that illustrate use of number

LANGUAGE ACTIVITIES

—Books related to mathematics—example, "The Wonderful World of Mathematics"

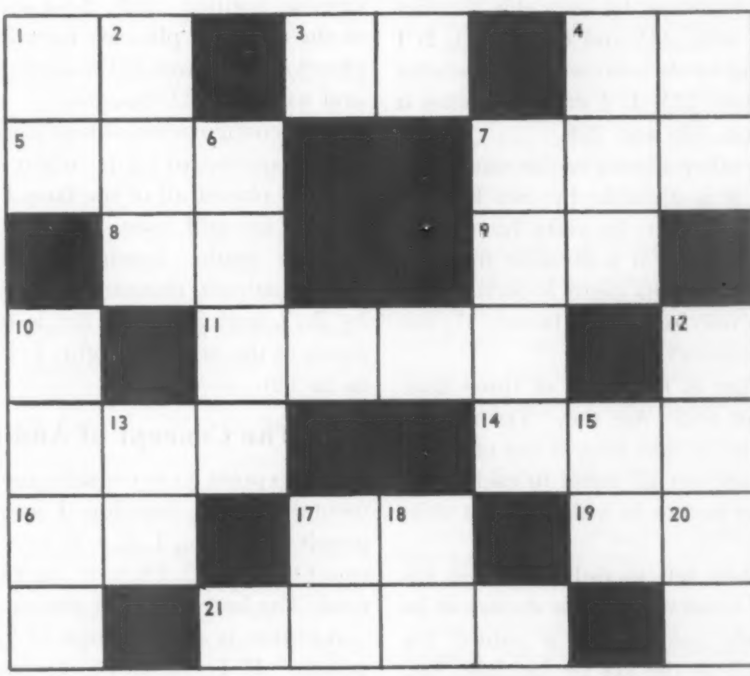
—Dramatizations of the social applications of mathematics

—Panels and debates on current issues and problems.

EDITOR'S NOTE. The Milwaukee Public Schools are to be commended for giving recognition in a professional and tangible way to the importance of in-service education in the teaching of arithmetic. The school administration and personnel also place emphasis on group guidance regarding ways of enriching the study of arithmetic for the gifted child by providing suggestions that teachers may follow. Other school systems may wish to make comparisons with practices in their own system and those of Milwaukee; or tell us wherein practices in their own system are better and why.

A CROSS-NUMBER PUZZLE FOR INTERMEDIATE GRADES

MARGARET F. WILLERDING
San Diego State College



ACROSS

1. 100 − 15
3. 3,240 ÷ 36
4. 7 + 8 + 9 + 6 + 5 + 3
5. 2,650 ÷ 25
7. 11,820 ÷ 60
8. 8 × 7
9. 315 ÷ 9
11. 52 × 92
12. 12 − 4
13. 8 × 8
14. 14 × 7
16. 7 × 7
17. 121 ÷ 11
19. $\frac{1}{2}$ of 164
21. From 10,272 subtract the year Columbus discovered America

DOWN

1. 9 × 9
2. 884 − 379
4. 2,765 ÷ 7
6. 3,356 + 3,288
7. 2,408 − 1,059
10. 63 ÷ 9
13. 780 − 711
15. 616 ÷ 7
16. 6 + 7 + 8 + 9 + 10
17. 2 × 4 + 9
18. Reverse the digits in the product of 9 × 9
20. 100 − 80

"Meaning" in Arithmetic

Jane M. Hill
Washington, D. C.

There are so many meanings in arithmetic that the only difficulty in writing on this subject is one of selection. The following ideas are miscellaneous rather than organized.

Probably the most fundamental concept of all is the concept of number. Let's take a number, say two, two, two (222). What do we know about this number?

If I were counting by ones this number would come after 221 and before 223. If I were counting by threes it would come after 219 and before 225. If I count by sixes it comes between 216 and 228.

There are other aspects to this number. I known that it is divisible by two because it is an even number; its right hand digit is even. I know that it is divisible by three because the sum of its digits is divisible by three. It has only one other factor, 37, another prime number.

This number is made up of three *twos*. They look just alike. Are they? Yes and no. They are alike in that each is *two* of something. They are not all equal to each other because of the *position* in which each is written.

Reading from left to right, the first *two* is valued ten times as much as the *two* to its right. And the middle *two* is valued ten times as much as the *two* on its right. Or, saying the same thing in reverse order, the two on the right is valued 1/10 of the two on its left, and the middle two, 1/10 of the two on its left. As it stands this number is made up of two hundreds, two tens, and two ones. The number is two hundred twenty-two.

Are three *twos* written in this way always two hundred twenty-two? Yes, but not always two hundred twenty-two of the same thing.

Suppose we put a decimal point here (22.2). We don't usually read it this way but this does represent 222 *tenths*. If we put the decimal point here (2.22) the number is then 222 *hundredths*. Shifting the point again to the left we have .222, 222 *thousandths*. Shift the point once more to the left and we have 222 *ten-thousandths*.

There is nothing to prevent our shifting the point to the right. Starting from the original position (222), if we shift the point to the right one place we have 222 *tens;* two places and we have 222 *hundreds;* three places and we have 222 *thousands*.

This configuration of *twos* has other interesting aspects. No matter where the decimal point is placed all of the facts I mentioned earlier are still true. All of these 222's, whether tenths, hundredths, thousandths, tens, hundreds, thousands—all are divisible by 2, 3, and 37. Each *two* is ten times as much as the *two* to its right; 1/10 of the *two* to its left.

The Concept of Addition

At this point let us consider another fundamental concept, addition. I may count the people in a room, 1, 2, 3, 4, · · · . Or I may count them 5, 12, 19, · · · , and so on to the total. The latter involves addition.

Addition is also a matter of "putting together." If I put three objects with five of the same kind of objects I have eight of these objects.

There are four rules or laws of addition. They have impressive names but the ideas involved are simple. By name they are:

1. Rule of likeness
2. Rule of compensation.
3. Law of commutation
4. Law of association.

All of you are familiar with the first—Only like quantities can be added. I shall come back to this in a moment.

Two other rules can be readily illustrated by the following:

$$
\begin{aligned}
42+36 &= (40+2)+(30+6) \\
&= 40+2+30+6 \qquad \text{(Association)} \\
&= 40+30+2+6 \qquad \text{(Commutation)} \\
&= (40+30)+(2+6) \quad \text{(Association)} \\
&= 70+8 \\
&= 78
\end{aligned}
$$

And a similar example will illustrate the rule of compensation:

$$37+59=37+60-1$$

That is, a sum is not changed if we increase one of the addends provided we decrease by the same amount.

I said I would come back to the rule of likeness. It is the most generally known. We see it in operation in the simplest problems and in the most complex. Why, in adding 5, 78, and 132 do we list them vertically with the right hand side straight? Would not a straight left edge make a better margin? And why, in adding decimal fractions, are the decimal points kept in a straight line? Why, in the addition of fractions must we get a common denominator? The answer in each instance is the same, the rule of likeness.

These rules of addition are fundamental not only to arithmetic, they are fundamental to mathematics. Addition is addition is addition—with integers, common fractions, decimal fractions, denominate numbers, signed numbers, literal numbers, radicals, and so on. The same rules and laws apply throughout. Addition is not changed though the concept has been broadened by more extensive application.

Meaning of Fraction

Finally, let us consider fractions, common fractions. The terminology of fractions is revealing. The word *fraction* itself is derived from the Latin word meaning "to break." The first concept of fraction was no doubt literally of something broken into parts—something less than one. In fact, when a fraction represents a quantity greater than one it is "improper." Is there anything really improper about 5/4?

A fraction has a *numerator* and a *denominator*. To numerate is to count. The *numerator* indicates "*how many*." We speak of *denominate numbers*. Do you see any relationships between the terms *denominator* and *denominate numbers?* I suspect that the dual use of the word was not accidental.

A fraction is a fascinating number. It has a distinctive characteristic which makes it so very useful. It can assume many forms without changing its value. Though its forms are unlimited in number the methods of changing the forms are only three.

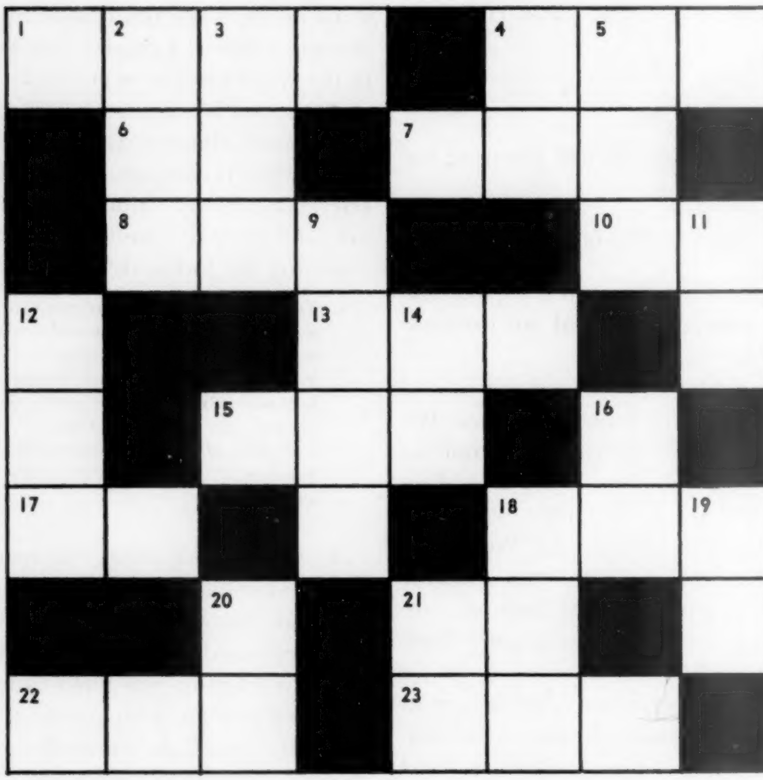
1. You can multiply the numerator and the denominator by the same number without changing the value of the fraction.
2. You can divide the numerator and the denominator by the same number without changing the value of the fraction.
3. You can divide the numerator by the denominator without changing the value of the fraction.

These then are some, but only a few, of the fundamental meanings in arithmetic. There are many other examples which could have been used. As I explained at the beginning, my selection was miscellaneous. Arithmetic is abundant with meaning. Arithmetic can be taught meaningfully. Arithmetic should be learned meaningfully.

EDITOR'S NOTE. The word "meaning" has been used a great deal in recent years in connection with the teaching of arithmetic. We must beware lest we develop too much of a restricted and specialized connotation with "meaning." The word "understanding" has honest and honorable meaning. We are at the stage in our literature where someone should establish greater clarity for *meaning, understanding, sense, significance*, and *insight* as these words are used in our discussions. If we accept "meaning" as a general classification into which other similar and related ideas are placed then it would be helpful if someone would classify and list the several categories of "meanings" which are desired in teaching and learning arithmetic. We have "meanings" of concepts, we have concepts of processes, we have understandings of these, etc. etc. Miss Hill has given elementary "meanings" of three items in arithmetic, there are many more. Most important is the fact that teachers are thinking about meaning and understanding and that suggests a style of teaching removed from rote memorization.

A CROSS-NUMBER PUZZLE FOR JUNIOR HIGH SCHOOL

Margaret F. Willerding
San Diego State College



ACROSS

1. How many yards are in 4,890 feet?
4. 15^2
6. 8 dozen
7. Write five minutes past eight in figures
8. How many acres are in one square mile?
10. XLIV represents what Hindu-Arabic numeral?
13. What is the total cost of 4 and $\frac{3}{4}$ yards of cambric at \$.40 a yard, 1 package of pins at \$.10, and 3 spools of thread at \$.05 each?
15. What is the average of 728, 964, 247, 425, and 316?
17. What is 90% of 100?
18. What is the perimeter of an equilateral triangle with a base of 104 feet?
21. What is the cost of 1 yard of bunting if 27 yards cost \$11.61?
22. How many quarters are there in \$33.75?
23. Write $4\frac{1}{2}$% as a decimal

Omit decimal points and percent signs in the puzzle. Just write the figures.

DOWN

2. What is the perimeter of a garden 180 feet by 168 feet?
3. Write the number of days in a regular year from January 1 through December 30
4. Express 1/5 as a per cent
5. Compute the interest for one year on \$127.00 at 2%
9. Reverse the digits in $\frac{1}{4}$ miles (express in feet)
11. What is the area in square feet of a rectangle 1 yard 2 feet by 9 feet?
12. What is the total cost, including 2% state sales tax, for 1 dozen cream puffs at \$.06 each, 2 loaves of bread at \$.17 a loaf, and 1 coffee cake at \$.40?
14. How many ounces are in one pound?
16. How many dozen cookies are needed to feed a troop of 33 Boy Scouts if each boy gets 4 cookies?
18. How many days remain in a normal year after January has passed?
19. What is the age in 1957 of a man born in 1930?
20. Write $\frac{3}{4}$ as a decimal
21. Write 2/5 as a decimal

A Self-Evaluation Study in Grade Seven

English Avenue School, Atlanta

UNDER THE CHAIRMANSHIP of L. E. Hambrick a committee of teachers took an honest look at their work in arithmetic during the school year 1956–1957 and decided to improve the arithmetic weaknesses within grade seven. Early in the study they listed the following weaknesses and then decided what they would do to correct them.

1. Lack of curiosity or wanting to learn for the sake of knowing instead of for a grade.
2. Lack of retention of fundamental processes.
3. Lack of recognition of the value of relationships in Arithmetic.
4. Lack of selfconfidence.
5. Lack of neatness and accuracy.
6. Failure to use everyday experiences in classroom situations.
7. Insufficient reading ability to determine what processes to use for a given problem.
8. Not enough skill in the fundamental processes.
9. Lack of comprehension.
10. Unfamiliarity with basic terms and/or concepts.

As the study group proceeded it was found that basic considerations were faced and these were studied under the sub-heads:

1. Philosophy—guiding principles
2. Objectives—general, specific, and emotional
3. Definition of the problem
4. Attack of the problem
5. Findings
6. Recommendations

Recommendations

1. In general all arithmetical teaching should follow this pattern:
   a. Teachers should seek a functional approach for introducing the new principle or process in a meaningful, concrete manner. Each fundamental process, for example, should be presented through the medium of concrete problems rather than abstract terms and examples.
   b. Teachers should use a simple, concise explanatory discussion, which furnishes essential background information for a sample problem involving the new principle.
   c. Teachers should use a sample problem or illustrative example, with a model solution and a detailed explanation of the solution.
   d. Teachers should use an exercise for immediate practice consisting of problems similar to the examples.
   e. A review of the problem at planned intervals to ensure retention of what the student has learned. This should be spaced reviews. This step by step sequential development of the subject matter enables the student to progress independently through his own efforts and with a minimum assistance from the teacher.
2. Make Arithmetic a functional thing. Use the project method where possible to cover the subject matter. A store, a bank, buying and selling or building a boat. If two or three projects of different types were carefully planned so that they would involve all phases of Arithmetic, one should have no difficulty.
3. That classes in Arithmetic be grouped according to specific criteria, such as abilities, needs, and interests.
4. That there should be a graduated balance between oral and written Arithmetic—life demands oral accuracy and speed in fundamental processes while specialized mathematics is largely written in nature.
5. That reasonable care should be taken to create in the pupil an active interest in contrast to a passive attitude (The pupil should attack his work with a feeling that it is his work, his learning and his responsibility.
6. That variations should be used to provide for individual students of different ability, insight, interest and time. A teacher should make relative suggestions as an effective method of leading his students to discover his potentialities and possibilities. Variations should be used for stimulating interest.
7. That a critical attitude be taught students about their own work and the work of others. (An ability to find and correct errors is of high social utility as well as an essential to surety of computation.)
8. That the vocabulary burden of Arithmetic presents a problem. Words not well understood should not be used in explanations. This warrants as specific teaching as any other aspect of Arithmetic does. Frequent grade-level tests should be made up or employed to find out the degree of vocabulary retention and of basic facts and relationships.
9. That all teachers should have a course in the use of supplementary aids, audiovisual aids, and multi-sensory aids for teaching Arithmetic.
10. A remedial program in Arithmetic should be employed as in the Reading Readiness Program to give meaning and understanding. The "drill

theory" and the "incidental theory" should be discarded for a more effective "meaning theory." (Remedial programs in Arithmetic make it possible to attain a major goal in education—to develop the child at his or her own rate and capacity. Remedial classes are the best TEMPORARY SOLUTION. The best final solution is to cover LESS MATERIAL, but cover it more thoroughly.)

11. For teachers in the instruction of Arithmetic, Arithmetic should be associated with concrete experiences and be set in problem situations which are based on the needs, interest,and abilities of the pupils.

Conclusion

A self-evaluation program should be continuous. If teaching and learning are to be improved and the needs of children served, constant attention must be given to the larger aims and goals. The seventh grade group of teachers in the English Avenue School at Atlanta have agreed to aim for the following with their pupils:

1. Give each child in our classroom laboratories some basic understanding of the number system.
2. Teach him to appreciate the value of abstract use of numbers in meeting the needs of life.
3. Give him the ingenuity and insight which will give to him interesting and meaningful relationships with life experiences.

INSPIRATION

Henry was like sediment in a pail of water. He had settled down in the Opportunity Room at the age of 9. Unless someone made an effort to remove him, Henry would stay. It was as good a place as any in which to dream and lead a passive existence.

No matter how Henry grew or the additional years his birthdays brought, his reading level remained 2.9. He had his better days when the teacher would silently breathe a prayer of thanks. Henry was improving! Next day he'd be back to 2.9.

Arithmetic held even less prominence in Henry's world. Difficulty probably best descibes his arithmetic achievements. Nothing ever made sense to Henry. Never give up! Maybe tomorrow he'll see the light. Try him again on measurements. But Henry was a past master at saying, "That don't make sense."

Strange things sometimes happen in the field of learning and so it was with Henry. One day the teacher was attempting to teach the girls in her class to use a sewing machine. They were having difficulty. They continually broke the thread. In desperation the teacher questioned, "Is there anyone here who can sew a fine seam?" Henry perked up and shyly said, "I can." He gave a fine demonstration and was duly praised for his achievements.

Henry was inspired. He could and would make his teacher an apron. There was one drawback. . . . Henry didn't recognize any form of measurement though he had been exposed many times to that form of arithmetic. Such teaching had been lost on Henry. He could try again? And he did. Together they worked, using all sorts of media from string to tape measure. They compared, measured again, recorded their findings and measured some more. Buying the material involved price, choice of material, thread, etc., and there was the problem of change making. Henry did all this on his way home from school.

In conclusion, Henry made the apron and he will never forget yards, feet, and inches. The moral, of course, even the least can learn something if the inspiration is there.

MARVEL STENBOL, *Lincoln School*
Charles City, Iowa

BOOKLET RECEIVED

A Handbook for Instructional Leaders on the Use of Encyclopedias in School, College of Education, University of Washington, Seattle 5, Washington. (Single copies may be obtained without cost.)

This handbook is intended to help teachers make more effective use of encyclopedias as instructional aids. The paragraph devoted to mathematics notes the use of encyclopedias for historical development of time, numbers, money, weights, and measures.

NCTM

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Minutes of the Annual Business Meeting

Bellevue-Stratford Hotel, Philadelphia, Pennsylvania

March 29, 1957

Dr. Howard F. Fehr, President, called the meeting to order at 4:00 P.M.

I. In his opening statement, Dr. Fehr expressed his appreciation to the membership for the fine co-operation he had received during the past year. He stated further that the Council was making fine progress in meeting its objectives, this progress being due to the excellent work of the members. In addition, Dr. Fehr made the following announcements:

A. *Guidance Pamphlet*—The National Academy of Sciences—National Research Council (Applied Mathematics Division) has proposed a brochure to go to high school students. This brochure will contain biographies of ten living mathematicians who are not teachers. In addition, it is to contain the biography of at least one person who is teaching mathematics. A further addition will be suggestions on what one must do in order to prepare for a career in mathematics. The Council and the M. A. of A., among other groups, have agreed to co-operate. The National Science Foundation has been approached for aid in publishing the pamphlet.

B. The U. S. Commission of Mathematics has selected two members of the Council to aid in interpreting our work to other nations. Those selected are: Dr. Howard F. Fehr and Dr. Henry W. Syer.

C. The National Association of Secondary School Principals has again invited the Council to supply the articles and to edit an issue of its publication, *The Bulletin.* This will be the March or April issue for 1959. The Board has accepted this invitation and will proceed to appoint an editor and staff.

D. *Yearbooks*—The schedule of future yearbooks as determined to date is as follows:

1. The 24th Yearbook, under the editorship of Dr. Phillip S. Jones, is due to be published in 1958. The title is: *Central Themes and Concepts.*
2. The 25th Yearbook, under the editorship of Dr. F. E. Grossnickle, is due to be published in 1959. The title is: *Arithmetic.*
3. The 26th Yearbook, under the editorship of Dr. Donovan A. Johnson, is due to be published in 1960. The title is: *Evaluation.*

II. Dr. F. L. Wren moved, seconded by Brother Francis Gerard, that the minutes of the April 13, 1956, meeting be approved as printed. The motion carried.

III. Dr. Houston T. Karnes, Recording Secretary, gave a brief review of the actions of the Board during the past year. This review included items which were thought to be of interest to the members. The review follows:

A. Dr. H. Glenn Ayre was appointed to fill the vacancy caused by the untimely death of Mr. G. E. Hawkins, as the Registered Agent for Illinois.

B. The National Science Foundation contributed $13,540.00 to be used in the development of the 24th *Yearbook* under the editorship of Dr. Phillip S. Jones. This *Yearbook*, on *Central Themes and Concepts*, is to be released in 1958.

C. Further plans were adopted for voting and for the counting of votes. They are as follows:

1. The membership will be given a one-month period in which to vote. The ballots must be postmarked no later than twenty-one days before the Annual Meeting.
2. An envelope arrangement will accompany the ballots so that the ballots may be returned in privacy. These envelopes will not be opened until the counting date, and then only by those who are employed to make the count. For this reason, checks, orders, and

the like should not be enclosed in the envelope with the ballot.

3. The Remington-Rand Company, or some similar agency with automatic tabulating machinery, will be employed to count the votes. This agency will submit three official copies of the results: one copy to the President, one copy to the Executive Secretary, and one copy to the Chairman of the Nominating Committee. These results will be considered as final.
4. As soon as the final report on the votes is received, the Executive Secretary will notify those who have been elected, after obtaining approval of his letter from the President and the Chairman of the Nominating Committee.

D. Places of future meetings:
1. The 1957 Summer Meeting will be held at Carleton College, Northfield, Minnesota. The dates are August 18–21, 1957.
2. As an experiment, there will be no Christmas Meeting for 1957.
3. The Annual Meeting for 1958 will be in Cleveland, Ohio.
4. The 1958 Summer Meeting will be held in Greeley, Colorado.
5. In view of the strong invitation from the New York City group and since the Council has never met in New York City, it was decided to have a 1958 Christmas Meeting in New York City.
6. The 1959 Annual Meeting will be held in Dallas, Texas.
7. The 1960 Annual Meeting will be held in Buffalo, New York. No further meetings are as yet scheduled. The President, however, invites all members and/or groups to extend invitations.

E. Due to the fine work of the Membership Committee, supported by other agencies of the Council, more than 2,000 new members have been added during the past year.

F. The first membership catalogue was published last fall. The decision of the Board to publish such a directory has been well received in view of the fine letters from the members. It has proved to be most useful to many people.

G. In view of increased costs the Board is considering an increase in dues. This will not be a hasty move and no increase will be considered until a complete study of the financial structure is made. This study will be made and presented to the Board at the Carleton College meeting this summer. Should an increase be warranted, it will not become effective until the summer or fall of 1958. The Council at the present has the lowest dues of any comparable organization.

H. Secondary Curriculum Committee; Mr. Frank Allen, Chairman. This committee has been expanded and its work accelerated. Definite plans have been formulated and approved for a three- or four-year study. The work of this committee is to be far-reaching. The membership of the Council can anticipate the report of this committee as being a major milestone in the progress of mathematical education.

I. Time does not permit a more detailed report of the actions of the Board during the past year. The report which you have heard contains those items which the Recording Secretary feels will be of major interest to you. The Secretary wishes it were possible to give a survey of all the fine committee reports which have been submitted to the Board. It should be said in passing, however, that these reports reveal that many interested and capable teachers of mathematics are doing excellent work in their particular areas and thus advancing the objectives of the Council on all fronts.

J. In the interest of correcting certain false statements and in creating a better understanding and to set forth the policy of the Council, the Board has adopted the resolution that follows. The resolution was prepared by a special committee composed of Dr. Myron Rosskopf, Dr. H. Vernon Price, and Mr. Robert E. K. Rourke, Chairman. The resolution will be read by Dr. Howard Fehr.

A resolution for The National Council of Teachers of Mathematics

PREAMBLE:

Recently, a spate of printed matter has unfolded shocking stories of the state of scientific education in this country. Newspapers, magazines, and reports of committees and commissions have told the public that, both as regards quality and quantity, our resources of teachers and students in mathematics and science are markedly inadequate. Some call the situation "a national emergency." Writers and speakers point out, with urgency and frequency, the need for increasing the supply of scientific manpower forthwith.

The undoubted seriousness of the situation has led many to attempt to search out its causes. Some of this searching has been objective and valuable, pointing out paths to worthwhile improvement; some of it, on the other hand, has resulted in distortions of facts and in fruitless fingerpointing. One of the most popular scapegoats is the high school—its curriculum and its teachers have faced criticism that is not always supported by the facts, or conducive to positive remedial action.

In particular, statistics are often used to make the picture blacker than it is. Percentage enrollments in algebra, geometry, and trigonom-

etry are sometimes given without any reference to the changed nature of the population since 1900; there is often no appreciation whatever of the problems of mass education. In 1900, a very large percentage of those in high school went on to college; in 1957, this is no longer the case. Pamphlet No. 118, issued by the U. S. Department of Health, Education, and Welfare, calls attention to some of these unfortunate misrepresentations.

"... It has been stated that only 1 out of 22 high-school students takes physics, whereas actually the ratio is closer to 1 out of 5. The number of pupils in chemistry has not declined 30% during the past 60 years—it has increased more than twentyfold. Two-thirds of the high-school pupils take algebra, instead of one-fourth."

"... Some surveys have compared the number of pupils in a particular course to the total enrollment in the school, rather than to the enrollment in the grade where the course is normally offered. On the former basis, if the course is a 12th-grade course and all 12th-graders take it, we would expect the percent to be about 19, since about 19% of all high-school pupils are in the 12th grade. This could be misinterpreted to mean that only 19% of the pupils who complete high school have taken the course."

In order to appreciate the nature of the problem, to understand something of its causes, and to take intelligent action toward its solution, certain facts must be faced:

A. The explosive development of mathematics since 1900 is probably without precedent in any other branch of learning, with the possible exception of physics. This embarrassment of mathematical riches has necessitated an up-dating in terms of subject matter for teachers at all levels.

B. The new developments in mathematics have been accompanied by a spectacular surge of new applications in the physical sciences, in the social sciences, in industry, and in national defense. Application rides hard on the heels of discovery. The requirements for those trained in mathematics today demand new goals, new content, new textbooks, and teachers with new training. A reshuffling of old materials will not suffice. A number of committees, commissions, and college staffs are now facing this fact frankly; the National Science Foundation is providing funds for a major attack on the problem.

C. The need for re-evaluation of aims, curriculum, methods, and teacher training is not only a problem for the high school; it demands attention from first grade right through college. The colleges are quite aware of this. A committee of the Mathematical Association of America reports that "there exists widespread dissatisfaction with the existing undergraduate program in mathematics." Already a number of colleges have developed courses radically different from the traditional program.

D. This great need for a better job of teaching mathematics at all levels places a special strain on those who must find the staff for the job. The graduate in mathematics formerly had little but teaching to which he might turn; now he has many vocational outlets. Opportunity no longer knocks just once, and often schools find it impossible to compete with industry for the services of well-trained graduates.

In view of the foregoing, it seems right and proper that the National Council of Teachers of Mathematics should at this time make known its awareness of the fact that the special needs of our time present special problems as yet unsolved. Since the Council has an important role to play in finding a solution to these problems, it should recall to the public its continued readiness to bear its full share in the task that faces all those responsible for the training of our scientists.

BE IT THEREFORE RESOLVED:

THAT The National Council of Teachers of Mathematics continue in the future, as in the past, to strive unceasingly for high professional standards for teachers of mathematics, as regards subject matter, professional training, and certification requirements.

THAT the National Council keep in close touch with other groups engaged in the task of improving the program and presentation of mathematics, and support the good work of these groups in every way possible.

THAT the National Council urge teachers of mathematics to take advantage of the many opportunities now available for in-service training —during the school year, in summer institutes, and, where possible, during leaves of absence.

THAT the National Council continue, with increased vigor, to improve the training of its membership through its *Yearbooks*, THE MATHEMATICS TEACHER, *The Arithmetic Teacher*, and its other publications.

THAT the National Council use every means at its disposal to encourage students to study mathematics as long as they are able to profit from it, realizing that quantity production without adequate quality will aggravate rather than ameliorate the situation.

THAT the National Council continue to pursue its study of goals and curriculum with a view to evolving and spelling out in detail programs of mathematical study that will challenge students of all levels of ability.

THAT the National Council urge its members to face squarely their special responsibilities to the very gifted in mathematics.

THAT the National Council work toward an ever-increasing co-operation between professional mathematicians, departments of education, and high school teachers of mathematics, in the firm belief that in such co-operation lies

our best hope of providing the scientifically trained personnel to meet the nation's needs.
IV. Mr. M. H. Ahrendt, Executive Secretary, gave a brief description of his work over the past year. He also gave a preliminary financial report. He stated that the final report would not be available until after the close of the fiscal year which ends on June 1. The completed report will be published. Mr. Ahrendt said the workload in the Washington Office was on the increase. In addition to the Executive Secretary, the staff is composed of five fulltime and one parttime employee. He said that, while the financial condition of the Council was sound, the expenses were beginning to exceed the income.
V. The 35th Annual Meeting breaks the attendance record. Dr. Fehr announced that the previous record of 1,322 registrants was made at the 28th Annual Meeting. The present registration figure for this, the 35th Meeting, is 1,343 with 150 more expected before the meeting comes to a close. Included in this number are: 399 from Pennsylvania, 178 from New Jersey, 140 from New York, 112 from Maryland, 18 from Canada, and one from Puerto Rico.
VI. Dr. F. L. Wren, Chairman of the Nomination and Election Committee, gave the following report of the recent balloting:

Vice-President for College: Dr. Robert E. Pingry
Vice-President for Junior High School: Miss Alice M. Hach
Directors: Dr. Clifford Bell
Mr. Robert E. K. Rourke
Miss Annie John Williams

VII. Dr. Fehr introduced the new officers and directors as declared elected by Dr. Wren.
VIII. Dr. Clifford Bell, Chairman of the new Committee on Nomination and Election, announced the plans of his Committee and encouraged the members to make recommendations. A box is located at the registration desk for the purpose of depositing recommendations. Two new vice-presidents are to be elected representing the elementary and secondary schools Three new directors are to be elected, one of whom must come from the Southwest district.
IX. Miss Ida May Bernhard presented the following resolution:

BE IT RESOLVED:

THAT the members of The National Council of Teachers of Mathematics express sincere appreciation and thanks to the Association of Teachers of Mathematics of Philadelphia and vicinity; the local committees; the co-operating high school students; the *Philadelphia Inquirer* and the *Evening Bulletin;* the wire services of the Associated Press, the United Press, the International News Service; the NEA Division of Press and Radio; and radio stations WPEN, WCAU, and WIP, which carried feature programs, all other stations carrying spot news and announcements; the Bellevue-Stratford Hotel; the staff in the Washington Office; and all others who shared in the planning and execution of this meeting in the city of Philadelphia. Their friendliness, co-operation, hospitality, and untiring efforts have made this a truly successful meeting.

After reading the resolution, Miss Bernhard moved and was seconded by Dr. H. Vernon Price that the resolution be adopted. The motion passed.
X. The meeting was duly adjourned at 5:00 P.M.

Respectfully submitted,
Houston T. Karnes,
Recording Secretary

Annual financial report

Edited by M. H. Ahrendt, Executive Secretary, NCTM, Washington, D.C.

Below is a brief financial report of the Council for the fiscal year ending May 31, 1957. The report shows that on June 1, 1956, we had total cash resources of $50,686.28. These resources had declined on May 31, 1957, to $42,569.20, making a net cash loss during the year of $8,117.08.

This report is more meaningful if viewed in the perspective of our financial picture over a period of years. Following is a record of our cash balances at the end of each of the six previous fiscal years.

May 31, 1951	$27,206.96
May 31, 1952	29,077.52
May 31, 1953	31,875.20
May 31, 1954	32,702.45
May 31, 1955	41,868.00
May 31, 1956	50,686.28
May 31, 1957	42,569.20

This record shows that the past year is the first in which we have not made a profit since we became affiliated with the NEA. Our loss during this year is the result of constantly increasing costs and of the broadening scope of our activities.

An example of increasing costs is the increase in printing and mailing costs of THE MATHEMATICS TEACHER and *The Arithmetic Teacher*. Examples of the broadening scope of our activities are the publication of our *Membership Directory* and our subsidizing of the important preliminary work of the Secondary School Curriculum Committee.

We seem to be faced with two alternatives: either increase our income by raising the dues, or drastically reduce certain phases of our program. Fortunately we have a sufficient cash reserve to tide us over a year of adjustment.

A few additional items in the report may need explanation. The net profit from the conventions is the balance left after all expenses, local and national, have been paid. The quotation from *Numbers and Numerals* was purchased by Simon and Schuster for use in their new publication entitled *The World of Mathematics*.

The expenses of the Washington Office included salaries for seven persons, supplies for both the office and the general program, handling and mailing of publications sold, renewal notices, membership and sales promotion, and travel by the Executive Secretary. The expenditures for the President's office included secretarial expenses, travel, and special projects. The expenditures for each journal include all costs for printing, mailing, editing, and travel.

Receipts and expenditures of The National Council of Teachers of Mathematics for the fiscal year, June 1, 1956-May 31, 1957

June 1, 1956—Total cash resources		$50,686.28
Receipts		
Memberships with THE MATHEMATICS TEACHER subscriptions	$ 28,690.89	
Memberships with *The Arithmetic Teacher* subscriptions	10,772.28	
Institutional subscriptions to THE MATHEMATICS TEACHER	14,889.15	
Institutional subscriptions to *The Arithmetic Teacher*	7,999.32	
Subscriptions to *The Mathematics Student Journal*	4,419.21	
Sale of advertising space in THE MATHEMATICS TEACHER	4,134.92	
Sale of advertising space in *The Arithmetic Teacher*	1,030.60	
Interest on U. S. Treasury Bonds	375.00	
Net profit from conventions	240.31	
Miscellaneous	145.32	
Sale of quotation from "Numbers and Numerals"	133.36	
Sale of publications		
Yearbooks	10,755.46	
Miscellaneous	10,279.66	
Total receipts	$ 93,865.48	
Expenditures		
Washington office	$ 32,992.05	
President's office	1,667.99	
Vice-President's office expenses	149.00	
THE MATHEMATICS TEACHER	29,064.55	
The Arithmetic Teacher	10,399.96	
The Mathematics Student Journal	2,876.95	
Committee work	721.34	
Secondary School Curriculum Committee	3,029.16	
Travel by Board members	2,188.19	
Preparation and printing of yearbooks	8,010.58	
Preparation and printing of supplementary publications	3,803.56	
Preparation, printing, and mailing of *Directory*	6,816.50	
Storage and shipment of publications, miscellaneous	262.73	
Total expenditures	$101,982.56	
Decrease in cash resources		$ 8,117.08
May 31, 1957—Total cash resources		$42,569.20